Neutrino Factory Linac RF cavity Transit Factor

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The Transit Factor (TF) is the ratio of the energy gain of a particle crossing a RF cavity to that of a particle crossing a perfect pillbox cavity with the same length and a uniform fixed maximum gradient. The first linac of a neutrino factory or muon collider must accelerate a bunch with a very large momentum spread from ~ 0.2 to 0.9 GeV/c. To design the linac's longitudinal optics, it is convenient to have a simple parameterization of the TF, so the results of `G4beamline` simulations using precalculated field maps were fit to a simple formula. The TF is <u>defined</u> here as:

$$TF \equiv (E_{out} - E_{in}) / (V_{max} * L_{cell})$$

For the NF single cell cavity, the length is taken to be 0.744 m, for the NF double cell cavity, it is taken to be twice that, 1.488 m. For both the maximum gradient is fixed at 23.09 MV/m.[1]

`G4beamline 2.08-kbb` is the standard release of `G4beamline 2.08`[2] with a modified pillbox routine that greatly improves the RF timing process; this author hopes that it will be incorporated into the next release. A standard neutrino factory cryomodule model with an on-axis μ^+ beam was used to determine the on-crest timing. Here the on-crest timing is *defined* as the timing that gives the maximum energy gain. For particles with $\beta<<1$, the distinction between finding the timing from maximum energy gain verses setting the time based on when the particle arrives in the middle of the cell (as used by earlier `G4beamline` versions) can be important.

The 3m cryomodule contains a counterwound solenoid and an single cell RF cavity "RF1"; since all the particles here are set to be on-axis, the solenoid plays no role in the tracking. The RF1 electric and magnetic field maps were provided by our colleagues at Imperial College[3] and converted into a format suitable for `G4beamline`.[4] Similarly, the 5m cryomodule also contains a solenoid and has a double cell RF cavity "RF2". The RF2 electric field map was also provided by our colleagues at Imperial College; while the magnetic field was estimated from the electric field using `txt2blfieldmap`. Again, by symmetry, the magnetic field plays no role in the on-axis tracking.

Many individual runs were made using momenta from 10 to 1000 MeV/c μ^+. The reference particle's output momentum was recorded by a virtual detector at the exit. Each run's results were placed into their own subdirectory, and the results were placed into a text spreadsheet. Columns added using `ooffice` to calculate the input total energy, the output total energy, and the transit factor.

The common `gnuplot`[5] program was used to plot and fit the data; a simple, but arbitrary, form was used to fit the data. The form of the expression has no physics content; it is merely a convenient

1 Interim Design Report, IDS-NF 020, http://www.ids-nf.org
2 http://g4beamline.muonsinc.com
3 Morteza Aslaninejad, Cristian Bontoiu Juergen Pozimski, Imperial College, UK – private communication
4 http://casa.jlab.org/external/MuonAccelerationGroup/MODELS/G4BL/TransitFactor/Sep2011/
5 http://www.gnuplot.info

parametrization. Only momenta from 100 MeV/c and greater was used for the fit.

$$f(p_{in}[\mathrm{MeV/c}]) \equiv e^{a/(p_{in}-b)^2-c}$$

cavity	a	b	c
RF1	3551.42 +/- 11.77	0.803546 +/- 0.186	0.613919 +/- 8.36e-05
RF2	3647.41 +/- 25.57	25.8256 +/- 0.309	0.615225 +/- 0.0002678

Table 1. Fitted TransitFactor coefficients in $TF(P_{in}) = \exp(a/(x-b)^2 - c)$.

The command files are in the appendix.

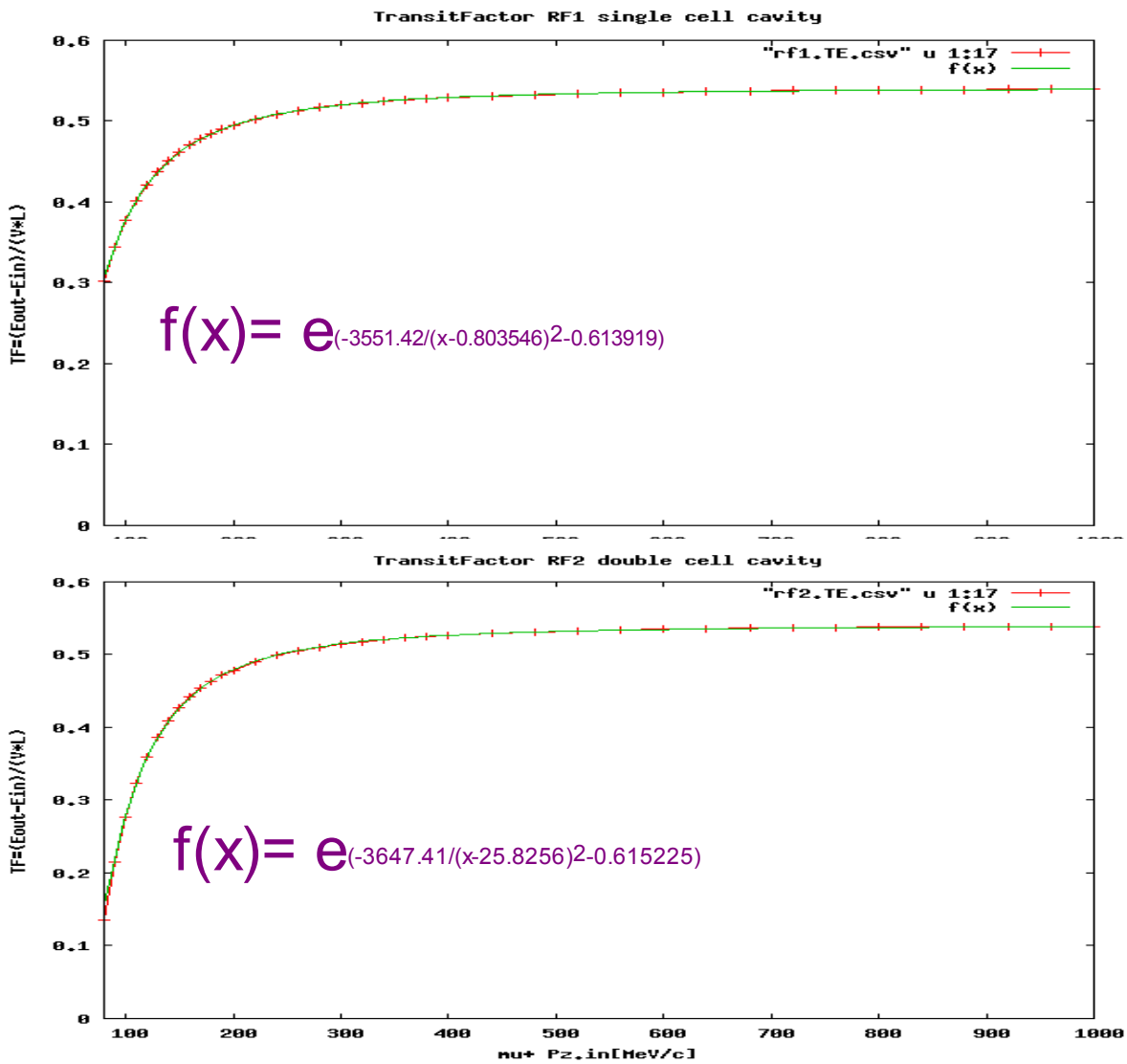


Figure 2a,b: Transit factors calculated by G4beamline (red) and fitted (green) for RF1 (a) and RF2 (b).

Appendix I. Field Maps

The RF1 field maps were 2 text files that were pasted together with the columns in the "x[m] y[m] z[m] Ex[V/m] Ey[V/m] Ez[V/m] Bx[T] By[T] Bz[T]" order. The `txt2blfieldmap`[6] program converted the units, normalizes the fields, and wrote the output into another text file appropriate for `G4beamline`. The command identified the columns, conversion factors, scaling factors, and frequency of the input text file:

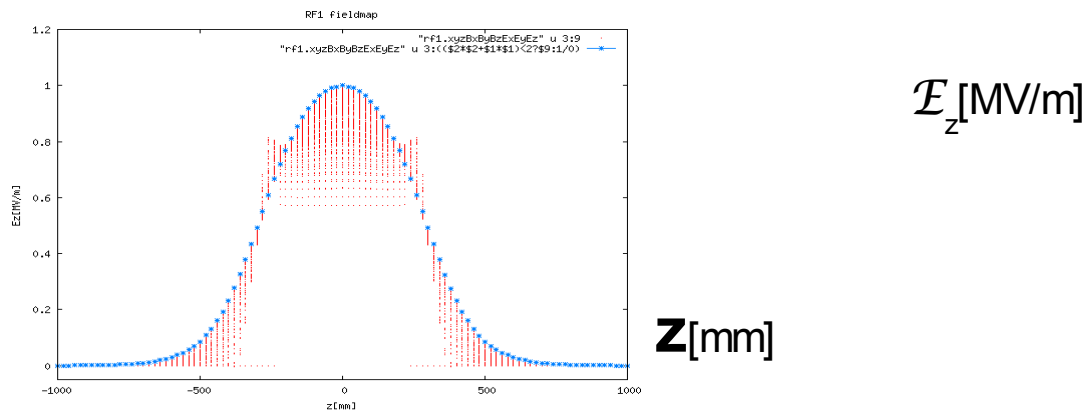


Figure 3a. RF1 electric field map, red: all points, blue" on-axis only.

```
$> txt2blfieldmap -i single_cell_EB.txt \
    -d "x y z Ex Ey Ez Bx By Bz" \
    -u 1000:1000:1000 -Bs 1. -Es 1.E-6 -RF 201.25 \
    +f "%.3f %.3f %.3f %.9e %.9e %.9e %.9e %.9e %.9e" \
    --comment "max|E|=1 MV/m" -o RF1_MV1.BLfmt -nT \
    ++normalize
```

The RF2 field map only contained the electric field, so the magnetic field was estimated from the curl of the electric field:

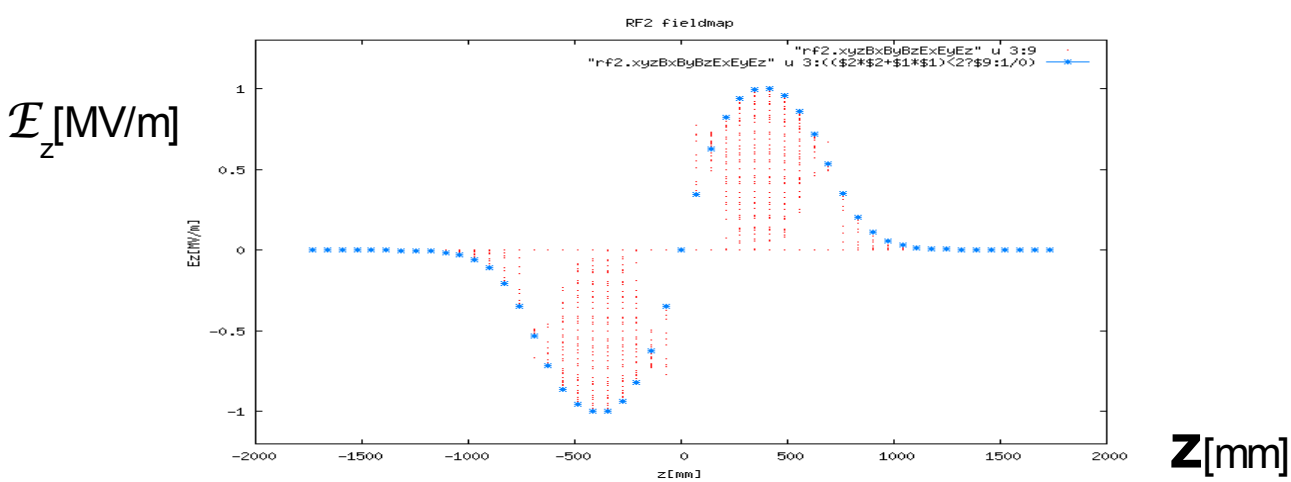


Figure 3b. RF2 electric field map, red: all points, blue" on-axis only.

6 /u/group/casa/acc_phys/6Dcooling/TOOLS/txt2blfieldmap/1.3/DOC/index.html

```
$>  txt2blfieldmap -i double_cell.dat \
    -d "x y z Ex Ey Ez"  -u 1000:1000:1000 -RF 201.25 \
    ++curlE2B +v ++normalize \
    +f "%.3f %.3f %.3f %.9e %.9e %.9e %.9e %.9e %.9e" \
    --comment "max|E|=1 MV/m, avg.quadrants, w/o time" \
    +aQ -nT -o RF2_MV1.v5.BLfmt
```

It is important to note that the time dependence of the field ought not be explicitly written at the end of the fieldmap; otherwise the sinusoidal RF time dependence would beat against the explicit time dependence inside a pillbox.

Single Cell Ez vs z on axis[3]

z[m]	Ez[V/m]	z[m]	Ez[V/m]	z[m]	Ez[V/m]
-1.000000	0.355961	-0.320000	437.090885	0.360000	326.175344
-0.980000	0.443904	-0.300000	494.530110	0.380000	277.104781
-0.960000	0.531846	-0.280000	554.129667	0.400000	233.519932
-0.940000	0.619789	-0.260000	612.185797	0.420000	193.036764
-0.920000	0.864888	-0.240000	668.526817	0.440000	160.590591
-0.900000	1.098216	-0.220000	722.464577	0.460000	129.945317
-0.880000	1.331544	-0.200000	771.098685	0.480000	108.186159
-0.860000	1.564872	-0.180000	815.782356	0.500000	86.427001
-0.840000	2.048865	-0.160000	855.893456	0.520000	70.483374
-0.820000	2.627053	-0.140000	891.575392	0.540000	57.171187
-0.800000	3.205242	-0.120000	921.229844	0.560000	45.154207
-0.780000	3.783431	-0.100000	946.175642	0.580000	37.701155
-0.760000	5.041976	-0.080000	968.435886	0.600000	30.248102
-0.740000	6.474551	-0.060000	982.066284	0.620000	23.067598
-0.720000	7.907126	-0.040000	995.393938	0.640000	19.335293
-0.700000	9.339701	-0.020000	999.965937	0.660000	15.602987
-0.680000	12.007709	0.000000	1004.224334	0.680000	11.870682
-0.660000	15.664806	0.020000	999.967759	0.700000	9.305726
-0.640000	19.321903	0.040000	995.711184	0.720000	7.878659
-0.620000	22.979000	0.060000	982.044937	0.740000	6.451591
-0.600000	30.466600	0.080000	968.751066	0.760000	5.024524
-0.580000	37.732713	0.100000	946.191124	0.780000	3.775261
-0.560000	44.998825	0.120000	921.210902	0.800000	3.197730
-0.540000	57.263269	0.140000	891.667245	0.820000	2.620199
-0.520000	70.415695	0.160000	855.904626	0.840000	2.042668
-0.500000	86.689013	0.180000	815.762370	0.860000	1.563082
-0.480000	108.285760	0.200000	771.080442	0.880000	1.330267
-0.460000	129.882508	0.220000	722.450242	0.900000	1.097453
-0.440000	161.175920	0.240000	668.457245	0.920000	0.864638
-0.420000	193.623838	0.260000	612.186524	0.940000	0.619450
-0.400000	234.282853	0.280000	554.093766	0.960000	0.531596
-0.380000	277.884757	0.300000	494.330201	0.980000	0.443742
-0.360000	327.230696	0.320000	436.211548	1.000000	0.355889
-0.340000	380.338156	0.340000	379.128440		

Double Cell Ez vs z on axis[3]

z[m]	Ez[MV/m]	z[m]	Ez[MV/m]
-1.7324999999999999	-0.0002879720000000	0.0693000000000000	1.3997299999999999
-1.6632000000000002	-0.0003740710000000	0.1385999999999999	2.5314999999999999
-1.5939000000000001	-0.0006329410000000	0.2079000000000000	3.3148399999999998
-1.5246000000000000	-0.0011836700000000	0.2772000000000000	3.7937099999999999
-1.4552999999999998	-0.0022807200000000	0.3465000000000000	4.0259099999999997
-1.3860000000000001	-0.0044312400000000	0.4157999999999999	4.0459300000000002
-1.3167000000000000	-0.0086246500000000	0.4851000000000000	3.8658700000000001
-1.2474000000000001	-0.0167953000000000	0.5544000000000000	3.4833400000000001
-1.1781000000000001	-0.0327099000000000	0.6237000000000000	2.8992399999999998
-1.1088000000000000	-0.0636294000000000	0.6929999999999999	2.1633599999999999
-1.0394999999999999	-0.1235010000000000	0.7623000000000001	1.4190799999999999
-0.9702000000000001	-0.2383570000000000	0.8316000000000000	0.8310510000000000
-0.9009000000000000	-0.4530100000000000	0.9008999999999999	0.4530100000000000
-0.8316000000000000	-0.8310510000000000	0.9702000000000001	0.2383570000000000
-0.7623000000000000	-1.4190799999999999	1.0394999999999999	0.1235010000000000
-0.6930000000000001	-2.1633599999999999	1.1088000000000000	0.0636294000000000
-0.6237000000000000	-2.8992399999999998	1.1781000000000001	0.0327099000000000
-0.5544000000000000	-3.4833400000000001	1.2473999999999998	0.0167953000000000
-0.4851000000000000	-3.8658700000000001	1.3167000000000000	0.0086246500000000
-0.4158000000000000	-4.0459300000000002	1.3860000000000001	0.0044312400000000
-0.3465000000000000	-4.0259099999999997	1.4552999999999998	0.0022807200000000
-0.2772000000000000	-3.7937099999999999	1.5246000000000000	0.0011836700000000
-0.2079000000000000	-3.3148399999999998	1.5939000000000001	0.0006329410000000
-0.1386000000000000	-2.5314999999999999	1.6632000000000002	0.0003740710000000
-0.0693000000000000	-1.3997299999999999	1.7324999999999999	0.0002879720000000
0.0000000000000000	-0.0000000000000000		

Appendix II. `G4beamline` model

The 3m and 5m NF cryomodules are complicated enough that sub-assemblies are described by macros. One macro constructs the solenoid, including its conductors, shielding, and field, while another a single cell RF cavity, including its material and fields and still another a double cell RF cavity. The walls of the cavity were approximated from the drawings, and the field maps were are described in Appendix I.

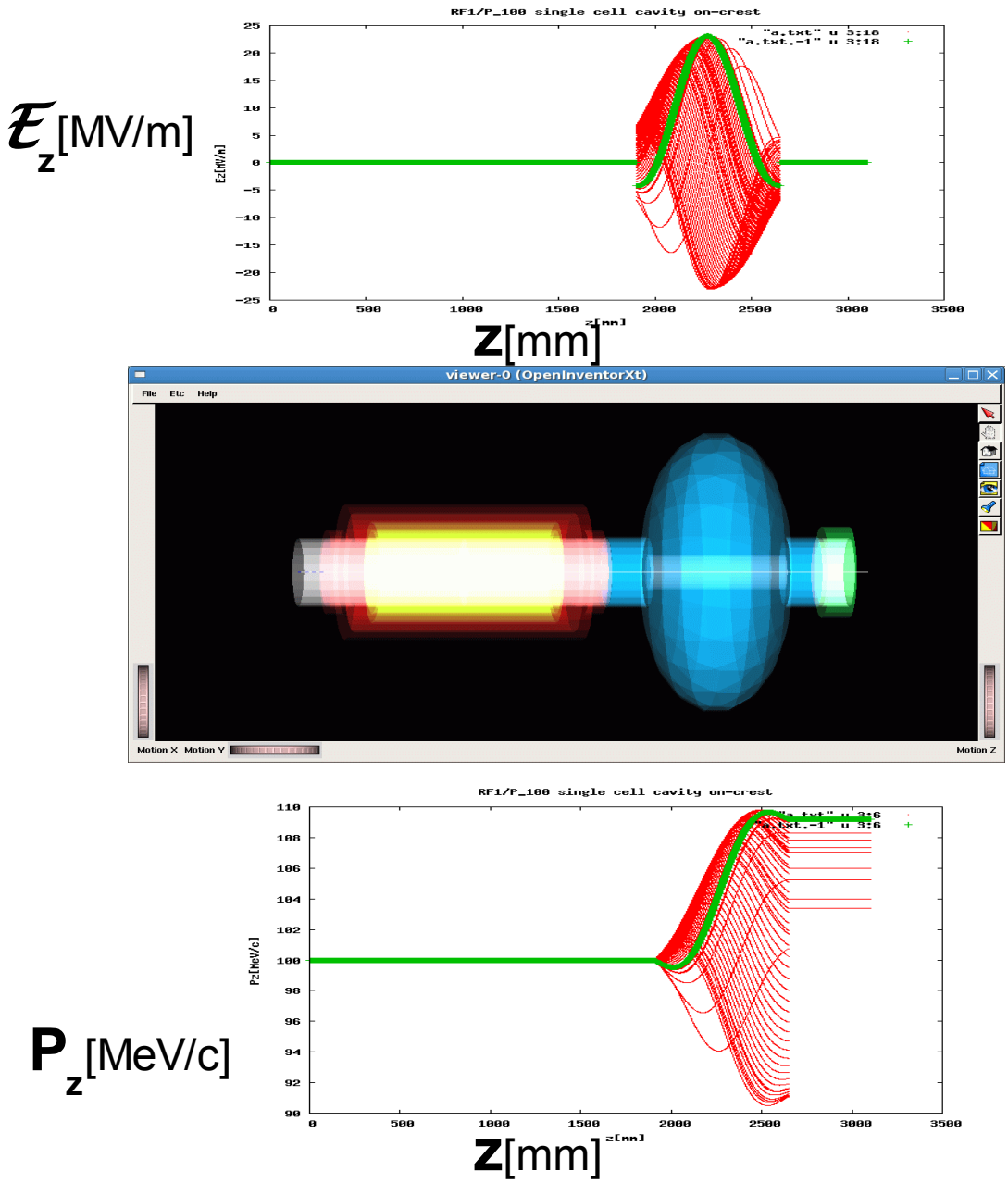


Figure 4a, b, c: The electric field as seen by the on-axis 100 MeV/c μ^+ reference particle while transversing the 3m cryomodule; the 3m cryomodule, and the momentum of the reference particle.

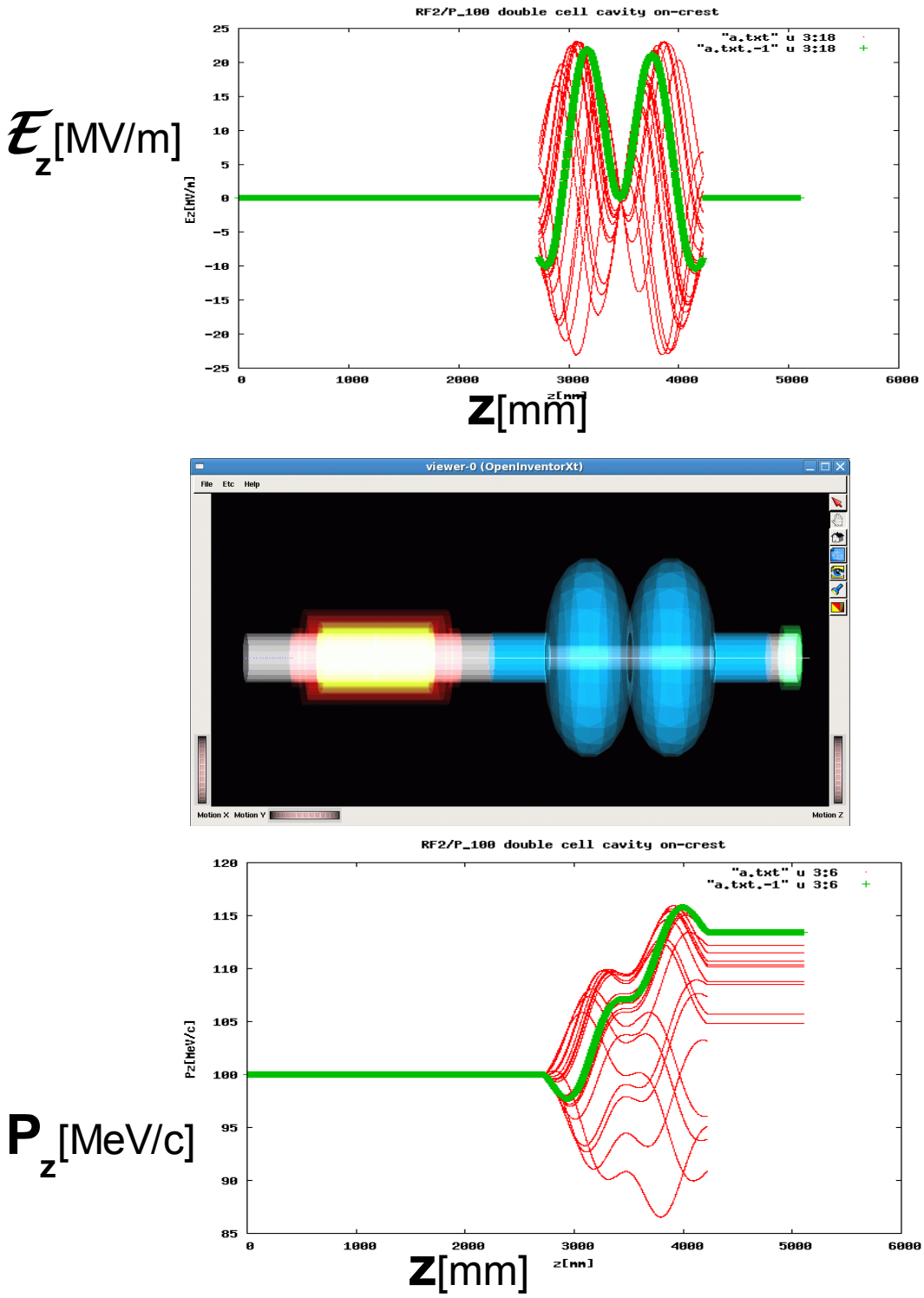


Figure 5a, b, c: The electric field as seen by the on-axis 100 MeV/c μ^+ reference particle while transversing the 5m cryomodule; the 5m cryomodule, and the momentum of the reference particle.

Fig. 4 displays the NF 3m cryomodule and the electric field and momentum for a 100 MeV/c μ^+ reference particle; the multiple iterative attempts used in the tuning are shown in red; the final on-crest

reference particle in green. Figure 5 displays the same for the NF 5m cryomodule.

For each input momentum from 10 to 1000 MeV/c, G4beamline was rerun and the output moved into a subdirectory, for example:

```
$> g4bl rf_1.in P0=100 > p100.log; mkdir P_100; mv -v  *.txt *.log P_100
```

The exit momentum for each reference particle were collected into spreadsheet; the energies and TF were calculated (Appendix IIIa and IIIb) and those results fitted to a simple expression using `gnuplot` (Appendix IV).

This simple exercise clearly demonstrates the advantage of `G4beamline`'s new pillbox routine when used with $\beta \ll 1$ particles, as seen by the runs with $10 < P_{in} < 100$ MeV/c.

Appendix IIIa. RF1 Spreadsheet

#Pinput	z	Pz	t	mass	E1	E2	TF=(E2-E1)/(L*V)
#MeV/c	mm	MeV/c	ns	MeV	MeV	MeV	-
10	3000	11.16	107.48	105.66	106.13	106.25	0.0068000
11	3000	12.07	97.69	105.66	106.23	106.35	0.0068000
12	3000	14.08	86.75	105.66	106.34	106.59	0.0148000
13	3000	11.85	86.43	105.66	106.46	106.32	-0.0078000
14	3000	17.02	73.14	105.66	106.58	107.02	0.0256000
15	3000	15.7	72.02	105.66	106.72	106.82	0.0058000
16	3000	14.49	69.48	105.66	106.86	106.65	-0.0125000
17	3000	18.08	63.18	105.66	107.02	107.19	0.0103000
18	3000	20.92	57.62	105.66	107.18	107.71	0.0308000
19	3000	19	58.06	105.66	107.35	107.35	0.0000000
20	3000	23	52.24	105.66	107.53	108.13	0.0349000
30	3000	34.07	35.67	105.66	109.83	111.02	0.0687000
40	3000	41.94	28.1	105.66	112.98	113.68	0.0408000
50	3000	52.6	23.08	105.66	116.89	118.03	0.0660000
60	3000	65.65	19.94	105.66	121.51	124.39	0.1682000
70	3000	77.37	17.83	105.66	126.74	130.96	0.2454000
80	3000	88.33	16.34	105.66	132.53	137.72	0.3022000
90	3000	98.88	15.24	105.66	138.79	144.71	0.3445000
100	3000	109.2	14.41	105.66	145.48	151.95	0.3768000
110	3000	119.39	13.75	105.66	152.52	159.43	0.4019000
120	3000	129.5	13.24	105.66	159.89	167.13	0.4218000
130	3000	139.56	12.82	105.66	167.52	175.04	0.4378000
140	3000	149.59	12.47	105.66	175.4	183.14	0.4508000
150	3000	159.6	12.19	105.66	183.48	191.41	0.4616000
160	3000	169.6	11.95	105.66	191.74	199.82	0.4705000
170	3000	179.6	11.74	105.66	200.16	208.37	0.4781000
180	3000	189.59	11.57	105.66	208.72	217.04	0.4845000
190	3000	199.58	11.42	105.66	217.4	225.82	0.4900000
200	3000	209.57	11.29	105.66	226.19	234.69	0.4948000
220	3000	229.54	11.08	105.66	244.06	252.69	0.5025000
240	3000	249.52	10.92	105.66	262.23	270.96	0.5085000
260	3000	269.49	10.79	105.66	280.65	289.46	0.5132000
280	3000	289.47	10.69	105.66	299.27	308.15	0.5170000
300	3000	309.46	10.6	105.66	318.06	327	0.5200000
320	3000	329.44	10.53	105.66	336.99	345.97	0.5225000
340	3000	349.43	10.48	105.66	356.04	365.05	0.5247000
360	3000	369.42	10.43	105.66	375.18	384.23	0.5264000
380	3000	389.41	10.39	105.66	394.42	403.48	0.5279000
400	3000	409.4	10.35	105.66	413.72	422.81	0.5292000
440	3000	449.38	10.29	105.66	452.51	461.64	0.5313000
480	3000	489.37	10.25	105.66	491.49	500.65	0.5329000
520	3000	529.36	10.21	105.66	530.63	539.8	0.5341000
560	3000	569.35	10.19	105.66	569.88	579.07	0.5351000
600	3000	609.35	10.16	105.66	609.23	618.44	0.5359000
640	3000	649.34	10.14	105.66	648.66	657.88	0.5366000
680	3000	689.34	10.13	105.66	688.16	697.39	0.5372000
720	3000	729.33	10.12	105.66	727.71	736.95	0.5376000
760	3000	769.33	10.11	105.66	767.31	776.55	0.5380000
800	3000	809.33	10.1	105.66	806.95	816.19	0.5383000
840	3000	849.33	10.09	105.66	846.62	855.87	0.5386000
880	3000	889.32	10.08	105.66	886.32	895.58	0.5389000
920	3000	929.32	10.08	105.66	926.05	935.31	0.5391000
960	3000	969.32	10.07	105.66	965.8	975.06	0.5392000
1000	3000	1009.32	10.07	105.66	1005.57	1014.84	0.5395000

Appendix IIIb. RF2 Spreadsheet

#Pinput	z	Pz	t	mass	E1	E2	TF=(E2-E1)/(L*V)
#MeV/c	mm	MeV/c	ns	MeV	MeV	MeV	-
10	5000	9.65	197.05	105.66	106.13	106.1	-0.0009000
11	5000	14.02	153.63	105.66	106.23	106.59	0.0103000
12	5000	15.46	139.41	105.66	106.34	106.78	0.0130000
13	5000	13.82	141.34	105.66	106.46	106.56	0.0030000
14	5000	14.69	131.39	105.66	106.58	106.67	0.0027000
15	5000	14.9	125.57	105.66	106.72	106.7	-0.0004000
16	5000	20.15	104.58	105.66	106.86	107.56	0.0204000
17	5000	15.37	111.85	105.66	107.02	106.77	-0.0072000
18	5000	19.2	100.4	105.66	107.18	107.39	0.0061000
19	5000	18.61	98.42	105.66	107.35	107.29	-0.0020000
20	5000	21.02	90.84	105.66	107.53	107.73	0.0056000
30	5000	34.9	58.36	105.66	109.83	111.27	0.0419000
40	5000	43.07	46.53	105.66	112.98	114.1	0.0327000
50	5000	53.96	38.29	105.66	116.89	118.64	0.0509000
60	5000	62.39	33.51	105.66	121.51	122.7	0.0348000
70	5000	72.47	29.81	105.66	126.74	128.12	0.0402000
80	5000	87.5	27.13	105.66	132.53	137.18	0.1355000
90	5000	101.05	25.22	105.66	138.79	146.2	0.2156000
100	5000	113.4	23.81	105.66	145.48	154.99	0.2770000
110	5000	124.96	22.73	105.66	152.52	163.64	0.3235000
120	5000	136.01	21.88	105.66	159.89	172.23	0.3593000
130	5000	146.74	21.2	105.66	167.52	180.82	0.3872000
140	5000	157.26	20.65	105.66	175.4	189.46	0.4093000
150	5000	167.63	20.19	105.66	183.48	198.15	0.4271000
160	5000	177.9	19.81	105.66	191.74	206.91	0.4415000
170	5000	188.09	19.48	105.66	200.16	215.74	0.4534000
180	5000	198.24	19.21	105.66	208.72	224.64	0.4634000
190	5000	208.35	18.97	105.66	217.4	233.61	0.4717000
200	5000	218.43	18.76	105.66	226.19	242.65	0.4788000
220	5000	238.54	18.42	105.66	244.06	260.9	0.4901000
240	5000	258.61	18.16	105.66	262.23	279.36	0.4986000
260	5000	278.64	17.96	105.66	280.65	298	0.5051000
280	5000	298.67	17.79	105.66	299.27	316.8	0.5103000
300	5000	318.68	17.65	105.66	318.06	335.74	0.5144000
320	5000	338.68	17.54	105.66	336.99	354.78	0.5177000
340	5000	358.68	17.44	105.66	356.04	373.92	0.5205000
360	5000	378.68	17.36	105.66	375.18	393.15	0.5228000
380	5000	398.68	17.3	105.66	394.42	412.44	0.5247000
400	5000	418.68	17.24	105.66	413.72	431.8	0.5263000
440	5000	458.67	17.14	105.66	452.51	470.68	0.5290000
480	5000	498.66	17.07	105.66	491.49	509.73	0.5310000
520	5000	538.66	17.02	105.66	530.63	548.92	0.5325000
560	5000	578.65	16.97	105.66	569.88	588.22	0.5337000
600	5000	618.65	16.93	105.66	609.23	627.6	0.5347000
640	5000	658.64	16.9	105.66	648.66	667.06	0.5355000
680	5000	698.64	16.88	105.66	688.16	706.58	0.5362000
720	5000	738.63	16.86	105.66	727.71	746.15	0.5367000
760	5000	778.63	16.84	105.66	767.31	785.77	0.5372000
800	5000	818.63	16.82	105.66	806.95	825.42	0.5376000
840	5000	858.63	16.81	105.66	846.62	865.1	0.5380000
880	5000	898.62	16.8	105.66	886.32	904.81	0.5383000
920	5000	938.62	16.79	105.66	926.05	944.55	0.5385000
960	5000	978.62	16.78	105.66	965.8	984.31	0.5388000
1000	5000	1018.62	16.77	105.66	1005.57	1024.09	0.5390000

Appendix IV. Scripts and Fitting

A simple Unix shell script illustrates the process of running the various momenta and gathering the results into a single text spreadsheet file. There are any number of ways this could be done, these are merely the author's preference for his own tools:[7]

```
# Transit Factor
# run each momentum [MeV/c]; store results in subdirectory
g4bl rf_1.in P0=100 > p100.log; mkdir P_100; \
                        mv -v  *.txt *.log P_100
...
g4bl rf_1.in P0=1000 > p1000.log; mkdir P_1000; \
                        mv -v  *.txt *.log P_1000
# erase any old files
rm -rf 1 2 3 rf1.csv
# make a numerically ordered list "1" of all P_# subdirectories
ls | grep P_ | sork > 1
touch 2;
# copy the 1st non-comment line from each exit* virtual detector file
# into file "2"
cat 1 | beshuffled -I \
     "cat #0/exit*.txt|no_bs -nc -nx|head -1 >> 2" | bash
# write the column header as a comment
echo \
  "#Pinput x y z Px Py Pz t PDGid EventID TrackID arentID Weight" \
     > rf1.csv
# take the input momentum from the name of the
# subdirectory and append the reference whole track point from the
# virtual detector
paste 1 2 | no_bs +S P_ >> rf1.csv
```

`OpenOffice`[8] was used to read the spreadsheet and add columns for E1, E2, and TF, and then save it as "rf1.TE.min.csv". Rather than do the fitting inside the spreadsheet, `gnuplot` was used with the function:

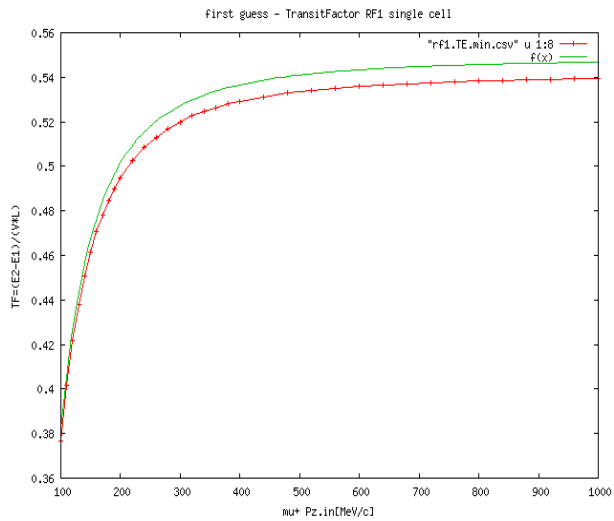
$$f(x) \equiv e^{a/(x-b)^2-c}$$

The `gnuplot` commands:

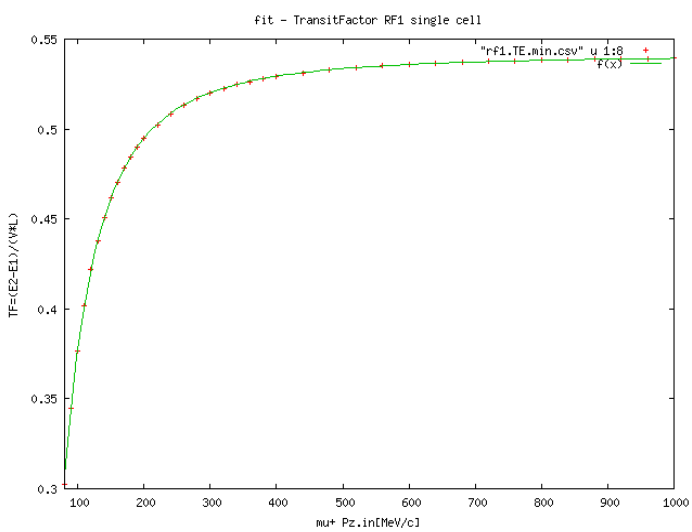
```
# define a function
f(x)= exp(a/(x-b)**2-c)
# important to set very good starting values
```

7 http://casa.jlab.org/internal/code_library/code_library.shtml
8 http://www.openoffice.org/

```
a=-3600
b=0.2
c=0.6
set xlabel "mu+ Pz.in[MeV/c]"
set ylabel "TF=(E2-E1)/(V*L)"
set title "first guess - TransitFactor RF1 single cell"
#column#1==Pin[MeV/c] column#8==TF[-]
plot [100:]"rf1.TE.min.csv" u 1:8 w lp,f(x)
pause 1
```




```
set title "fit - TransitFactor RF1 single cell"
#only consider points with Pin>100 MeV/c
fit [100:] f(x) "rf1.TE.min.csv" u 1:8 via a,b,c
# results written to "fit.log"
plot [80:]"rf1.TE.min.csv" u 1:8,f(x)
```



The same process was used for RF2.