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Configuration of the Muon Collider/Neutrino Factory Front End

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1. Introduction

The Muon Collider/Neutrino Factory muon front end consists of a pion decay channel and longitudinal drift, followed by an adiabatic buncher, phase rotation system and ionization cooling lattice.

The present design is based on the lattice presented in the Neutrino Factory Study 2A report [1] and subsequently the International Scoping Study [2] with several modifications: the taper from the target solenoid has been adjusted; the drift, buncher and phase rotation solenoid field strength has been reduced from 1.75 T to 1.5 T; the whole system has been shortened; and the thickness of lithium hydride absorbers in the cooling section has been increased. These changes result in the same muon capture performance in a shorter bunch train, reducing requirements on some systems downstream of the muon front end.

2. Decay and Longitudinal Drift

Downstream of the target solenoid the magnetic field is adiabatically reduced from 20 T to 1.5 T over a distance of 15 m while the beam aperture pipe radius increases from 0.075 m to 0.3 m. This arrangement captures within the 1.5 T decay channel a secondary pion beam with a large energy spread.

The initial proton bunch is relatively short (between 1 and 3 ns depending on the proton driver design) resulting in a short pion bunch. As the secondary pions travel from the target they drift longitudinally, following $ct(s) = s/\beta_z + ct_0$, where s is distance along the transport line and $\beta_z = v_z/c$ is the relativistic longitudinal velocity. Hence, downstream of the target, the pions and their daughter muons develop a position-energy correlation in this RF-free decay channel. In the baseline, the longitudinal drift length $L_D = 57.7$ m, and at the end of the decay channel there are about 0.4 muons of each sign per incident 8 GeV proton.

3. Buncher

The drift channel is followed by a buncher section that uses RF cavities to form the muon beam into a train of bunches, and a phase-energy rotating section that decelerates the leading high energy bunches and accelerates the late low energy bunches, so that each bunch has the same mean energy. The design delivers a bunch train that is less than 80 m long. This is an improvement over previous versions of the design developed for the ISS [2] which delivered a 120 m long bunch train containing the same number of muons.

A shorter bunch train may make some downstream systems easier to design. For example, one of the constraints on the minimum length of the decay rings is the total length of the bunch train. By making the bunch train shorter, it may be possible to make the decay rings shorter. Also, the FFAG ring has a rather demanding kicker system, mostly driven by the total circumference of the ring but also influenced by the bunch train length. A shorter bunch train may make these kickers easier to construct.

To determine the required buncher parameters, we consider reference particles (0, N_B) at $p_0 = 233$ MeV/c and $p_{N_B} = 154$ MeV/c, with the intent of capturing muons from an initial kinetic energy range of 50 to 400 MeV. The RF cavity frequency f_{RF} and phase are set to place these particles at the center of bunches while the RF voltage increases along the transport. These conditions can be maintained if the RF wavelength λ_{RF} increases along the buncher, following

$$N_B \lambda_{RF}(s) = N_B \frac{c}{f_{RF}(s)} = s\left(\frac{1}{\beta_{N_B}} - \frac{1}{\beta_0}\right) \tag{1}$$

where s is the total distance from the target, β_0 and β_{N_B} are the velocities of the reference particles, and N_B is an integer. For the present design, N_B is chosen to be 10, and the buncher length is 31.5 m. With these parameters, the RF cavities decrease in frequency from 320 MHz ($\lambda_{RF} = 0.94$ m) to 230 MHz ($\lambda_{RF} = 1.3$ m) over the buncher length.

The initial geometry for RF cavity placement uses 0.4–0.5 m long cavities placed within 0.75 m long cells. The 1.5 T solenoid focusing of the decay region is continued through the buncher and the following rotator section. The RF gradient is increased from cell to cell along the buncher, and the beam is captured into a string of bunches, each of them centered about a test particle position, with energies determined by the spacing from the initial test particle such that the i^{th} reference particle has velocity

$$1/\beta_i = 1/\beta_0 + \frac{i}{N_B}\left(\frac{1}{\beta_{N_B}} - \frac{1}{\beta_0}\right). \tag{2}$$

In the initial design, the cavity gradients V_{RF} follow a linear increase along the buncher,

$$V_{RF}(s) \approx 9\frac{z}{L_B} MV/m \tag{3}$$

where z is distance along the buncher. The gradient at the end of the buncher is 9 MV/m. This gradual increase of the bunching voltage enables a somewhat adiabatic capture of the muons into separated bunches, which minimizes phase-space dilution.

In the practical implementation of the buncher concept, this linear ramp of cavity frequency is approximated by a sequence of RF cavities that decrease in frequency along the 33 m beam transport allotted to the buncher. A total of 37 RF cavities are specified, with frequencies varying from 319.6 to 233.6 MHz, and RF gradients from 4 to 7.5 MV/m. The number of different RF frequencies is limited to a more manageable 13 (1-3 RF cavities per frequency). The linear ramp in gradient described by equation (3) is approximated by the placement and gradient of the cavities in the buncher. Table I shows a summary of the RF cavities that are needed in the buncher, rotator and cooling sections.

	Length [m]	Number of cavities	Frequencies [MHz]	Number of frequencies	Peak gradient [MV/m]	Peak power requirements
Buncher	33	37	319.6 to 233.6	13	4 to 7.5	1–3.5 MW/freq.
Rotator	42	56	230.2 to 202.3	15	12	2.5 MW/cavity
Cooler	75	100	201.25	1	15	4 MW/cavity
Total	240 m	193	319.6 to 201.25	29	1000 MV	550 MW

TABLE I. Summary of front end RF requirements.

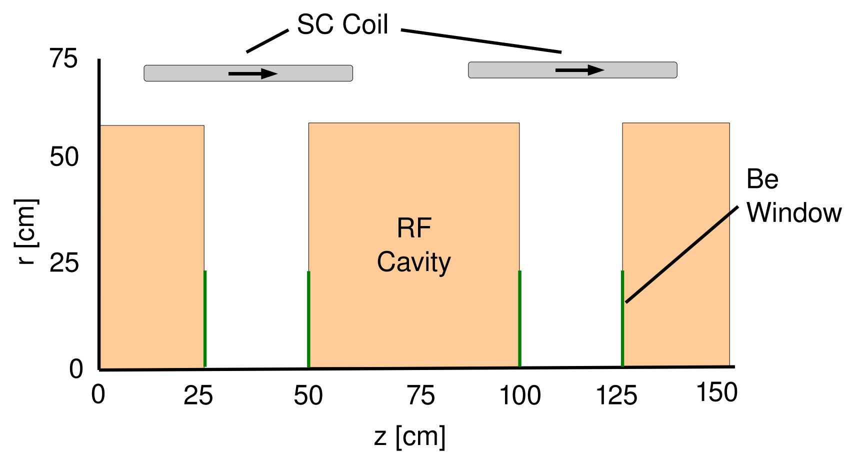

FIG. 1. Schematic radial cross section of a rotator cell.

4. Rotator

In the rotator section, the RF bunch spacing between the reference particles is shifted away from the integer N_B by an increment δN_B, and phased so that the high-energy reference particle is stationary and the low-energy one is uniformly accelerated to arrive at the same energy as the first reference particle at the end of the rotator. We impose $\delta N_B = 0.05$ and the bunch spacing between the reference particles is $N_B + \delta N_B = 10.05$. This is accomplished using an RF gradient of 12 MV/m in 0.5 m long RF cavities within 0.75 m long cells. The RF frequency decreases from 230.2 MHz to 202.3 MHz along the length of the 42 m long rotator region. A schematic of a rotator cell is shown in Fig. 1.

The RF frequency is set by requiring that the reference particle trajectories be spaced in ct by $(N_B + \delta N_B)$ wavelengths. In practical implementation, a continuous change in frequency from cavity to cavity is replaced by grouping adjacent sets of cavities into the same RF frequency. The 42 m long RF rotator then contains 56 RF cavities grouped within 15 frequencies.

Within the rotator, as the reference particles are accelerated to the central energy (at $p = 233$ MeV/c) at the end of the channel, the beam bunches formed before and after the central bunch are decelerated and accelerated respectively, obtaining at the end of the rotator a string of bunches of equal energy for both muon species. At the end of the rotator the RF frequency matches into the RF frequency of the ionization cooling channel (201.25 MHz). The average momentum at the rotator is 230 MeV/c. The performance of the bunching and phase rotation channel, along with the subsequent

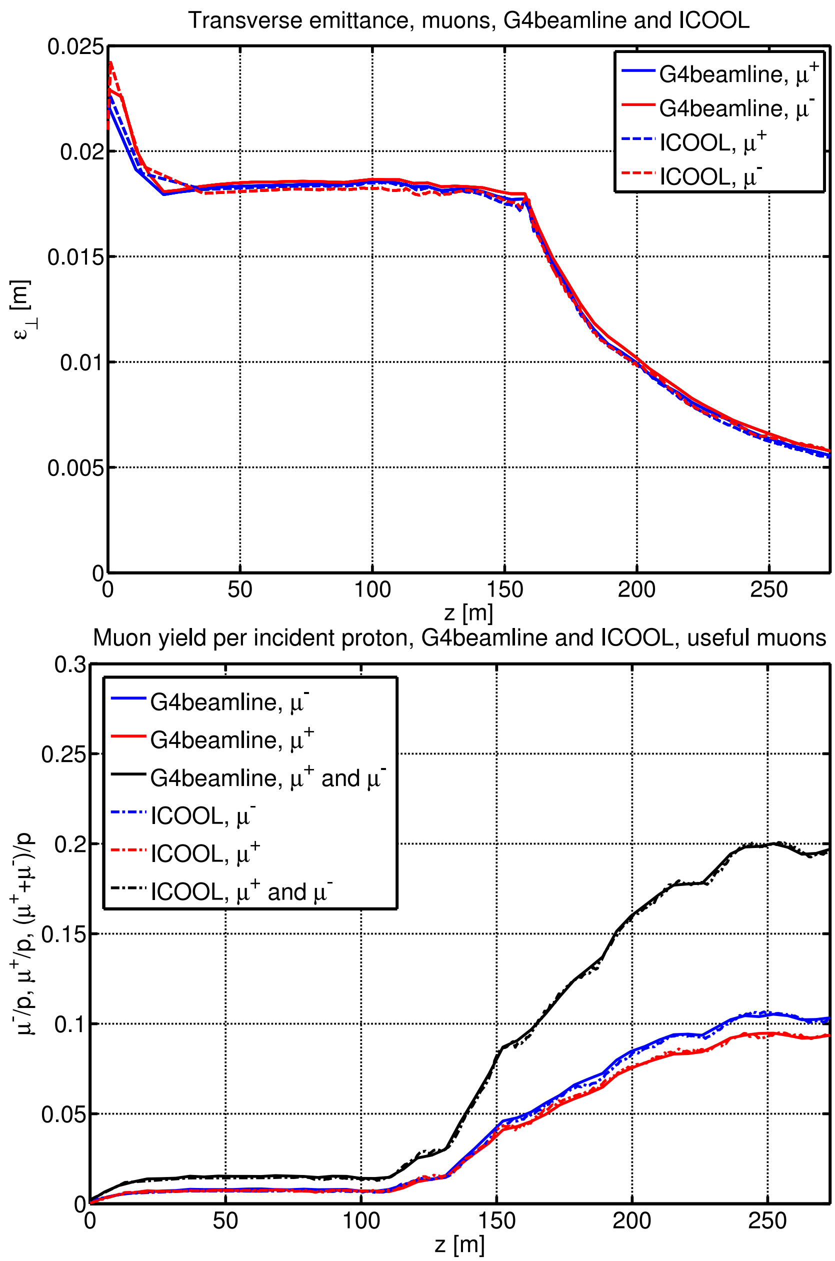

FIG. 2. Performance of the bunching and cooling channel as a function of distance along the channel, as simulated using the ICOOL code [3] and the G4Beamline code [4]. (top) The evolution of the RMS transverse emittance. (bottom) The evolution of the number of muons within a reference acceptance (muons within 201.25 MHz RF bunches with momentum in the range 100–300 MeV/c, transverse amplitude squared less than 0.03 m and longitudinal amplitude squared less than 0.15 m). The cooling section starts at $s = 155$ m, where the RMS transverse emittance is 0.018 m and 0.08 μ per proton are in the reference acceptance. The capture performance is shown for a cooling channel extending to $s = 270$ m although in this design the cooling channel extends only to 230 m. Acceptance is maximal at 0.20 μ per initial proton at $s = 240$ m (85 m of cooling) and the RMS transverse emittance is 0.007 m. At $s = 230$ m (75 m of cooling) the number of μ per proton is 0.19 and the transverse emittance is 0.0075 m.

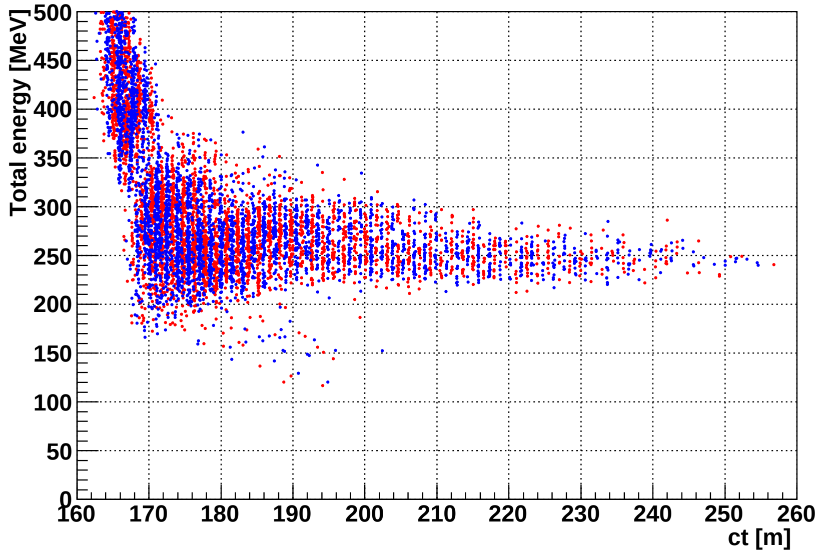

FIG. 3. Distribution of particles in longitudinal phase-space at the phase rotation end. $\mu+$ are shown in red and $\mu-$ are shown in blue.

cooling channel, is displayed in Fig. 2, which shows, as a function of the distance down the channel, the number of muons within a reference acceptance. The phase rotation increases the "accepted" muons by a factor of 4.

A critical feature of the muon production, collection, bunching and phase rotation system is that it produces bunches of both signs ($\mu+$ and $\mu-$) at roughly equal intensities. This occurs because the focusing systems are solenoids which focus both signs, and the RF systems have stable acceleration for both signs, separated by a phase difference of π. The distribution of muons in longitudinal phase space for particles of both signs at the end of the rotator is shown in Figure 3.

5. Cooling Channel

The baseline cooling channel design consists of a sequence of identical 1.5 m long cells (Fig. 4). Each cell contains two 0.5 m-long RF cavities, with 1.1 cm thick LiH discs at the ends of each cavity (4 per cell) and a 0.25 m spacing between cavities. The LiH discs provide the energy loss material for ionization cooling. The cells contain two solenoidal coils with opposite sign currents. The coils produce an approximately sinusoidal variation of the magnetic field in the channel with a peak value on-axis of 2.8 T, providing transverse focusing with $\beta_{\perp} = 0.8$ m. The currents in the first two cells are perturbed from the reference values to provide matching from the constant-field solenoid in the buncher and rotator sections. The total length of the cooling section is 75 m (50 cells). Based on the simulation results shown in Fig. 2, the cooling channel is expected to reduce the RMS transverse normalized emittance from $\epsilon_N = 0.018$ m to $\epsilon_N = 0.0075$ m. The RMS longitudinal emittance is $\epsilon_L = 0.07$ m/bunch.

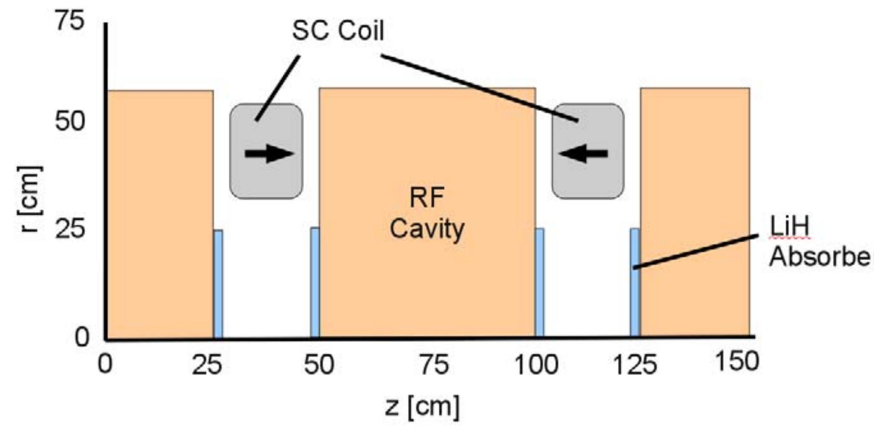

FIG. 4. Schematic radial cross section of a cooling cell.

The effect of the cooling can be measured by counting the number of simulated particles that fall within a reference acceptance that approximates the expected acceptance of the downstream accelerator.

The squared amplitude $A_\perp^2$ is given by

$$A_\perp^2 = p_z/m\left[\beta_\perp(x'^2 + y'^2) + \gamma_\perp(x^2 + y^2) + 2\alpha_\perp(xx' + yy') + 2(\beta_\perp\kappa - \mathcal{L})(xy' - yx')\right] \tag{4}$$

where $\beta_\perp$, $\alpha_\perp$, $\gamma_\perp$ are solenoidal equivalents of the Twiss parameters, κ is the solenoidal focussing strength and $\mathcal{L}$ is the dimensionless kinetic angular momentum [5, 6].

For longitudinal motion, the variables $t_c = ct$ (phase lag in periods within a bunch multiplied by RF wavelength) and ΔE (energy difference from centroid) are used rather than (z, z'). The longitudinal squared amplitude is given by

$$A_L^2 = \frac{c}{m_\mu}\left[\frac{t_c^2}{\delta} + \delta\left(\Delta E - \frac{\alpha_L t_c}{\delta}\right)^2\right] \tag{5}$$

where δ is defined by

$$\delta = \frac{c\left\langle t_c^2\right\rangle}{m_\mu \epsilon_L}, \tag{6}$$

ϵ_L is a normalized longitudinal emittance

$$\epsilon_L = \frac{c}{m_\mu}\sqrt{\left\langle t_c^2\right\rangle\left\langle \Delta E^2\right\rangle - \left\langle t_c \Delta E\right\rangle^2} \tag{7}$$

and α_L is a correlation factor

$$\alpha_L = \frac{c}{m_\mu \epsilon_L}\left\langle t_c \Delta E\right\rangle. \tag{8}$$

Following criteria developed using the ECALC9 program, a particle is considered to be within the acceptance of the machine if the transverse amplitude squared $A_\perp^2$ is less than 0.03 m and the longitudinal amplitude squared is less than 0.15 m. Note that the transverse and longitudinal notations are not the same, and transverse-longitudinal amplitude correlations are not included.

This is a first approximation to the muon accelerator acceptance, and it is used in the present tables for consistent comparisons of simulations.

Using the output from our re-optimized buncher and rotator, we have tracked particles through the cooling channel, and obtain within the reference acceptances 0.20 μ per 8 GeV incident proton. The acceptance criteria remove larger amplitude particles from the distribution, and the RMS emittance of the accepted beam is therefore much less than that of the entire beam. The RMS transverse emittance $\epsilon_\perp$ of the accepted beam is 0.004 m and the RMS longitudinal emittance is 0.036 m.

At the end of the cooling channel there are interlaced trains of positive and negative muon bunches. The trains of usable muon bunches are 80 m long (50 bunches), with 70% of the muons in the leading 20 bunches (30 m). The bunch length is 0.16 m in ct for each bunch with a mean momentum of 230 MeV/c and an RMS width δp of 28 MeV/c. For the accepted beam, the RMS bunch width is 3.8 cm, and the RMS transverse momentum is 10 MeV/c.

6. Simulation Codes

Two independently-developed codes have been used for tracking simulations of the muon front end by the Monte Carlo method.

- ICOOL version 3.20
- G4Beamline version 2.06

ICOOL [3] is under active development at Brookhaven National Laboratory. It is a 3-dimensional tracking program that was originally written to study ionization cooling of muon beams. The program simulates particle-by-particle propagation through materials and electromagnetic fields. The physics model is most accurate for muons in the kinetic energy range 50–1000 MeV, but tracking of electrons, pions, kaons, and protons is also possible. ICOOL includes a number of custom models for particle decay, delta ray production, multiple Coulomb scattering, ionization energy loss and energy straggling.

G4beamline [4] is a particle tracking and simulation program under active development by Muons Inc. Physics processes are modelled using the Geant4 toolkit with the QGSP_BERT physics package [7] and it is specifically designed to easily simulate beamlines and other systems using single-particle tracking.

Both codes use semi-analytic procedures to compute fields. Solenoid fields are generated as a sum of elliptic integrals calculated using the solenoid coil geometry [8]. RF cavities are modelled using a Bessel function radially and a sinusoid in t for the ideal field produced by a cylindrical pillbox cavity.

Good agreement is shown in the muon yield and the yield of other particle species from the two codes. Note that different versions of ICOOL have been shown to disagree at the level of a few % [9].

Analysis of results is performed using ecalc9f version 2.07 in addition to custom scripts written in Python and MATLAB. The beam has been generated using MARS 15.07. We expect a significant systematic error owing to uncertainty in the models used to generate the input beam.

7. RF Requirements and Design

The RF cavities in this design are all normal conducting cavities having 29 frequencies in the range 201.25 MHz to 320 MHz. The cavities are 50 cm long with peak field gradients in the range 4 to 15

MV/m, with the highest voltage required for the 201 MHz cavities. The power consumption of these cavities has been estimated semi-analytically using standard formulae, and the results are listed in Tables II and III, together with the RF cavities required. The position and phase of every cavity is listed in Table IV

Frequency [MHz]	Voltage (per frequency) [MV]	Number of cavities	Length [m]	Gradient [MV/m]	Peak RF Power (per frequency) [MW]
Buncher					
319.63	1.37	1	0.4 m	4 MV/m	0.2
305.56	3.92	2	0.4	5	0.6
293.93	3.34	2	0.45	4	0.5
285.46	4.8	2	0.45	5.5	1
278.59	5.72	2	0.45	6.4	1.25
272.05	6.66	3	0.45	5	1.5
265.8	7.57	3	0.45	5.7	1.5
259.83	8.48	3	0.45	6.5	2
254.13	9.41	3	0.45	7	2.3
248.67	10.33	4	0.45	5.7	2.3
243.44	11.23	4	0.45	6.5	2.5
238.42	12.16	4	0.45	7	3
233.61	13.11	4	0.45	7.5	3.5
Total	98.1	37			22

TABLE II. Front end RF requirements for the buncher system.

Several RF cavities have been constructed to support the muon accelerator design effort [10]. A 43 cm long, 201 MHz RF cavity has been constructed and operated at peak field gradients up to 21 MV/m and 8 more RF cavities are under construction as part of the Muon Ionization Cooling Experiment. Additionally several 805 MHz cavities have been constructed and operated at gradients up to 40 MV/m. Design and construction of cavities with intermediate frequencies is not expected to present any additional difficulties.

8. Effect of Magnetic Field on RF Gradient

There is some empirical evidence that suggests that magnetic fields overlapping RF cavities, as present in the Neutrino Factory front end, may induce breakdown in the cavities [11, 12]. The performance of the muon front end using a reduced field has been explored using ICOOL. In Fig. 5, the muon transmission is shown as a function of fractional change in RF gradient in the buncher, rotator and cooling. The simulation indicates that around the nominal gradient, muon transmission is rather insensitive to peak achievable gradients. If the achievable RF gradient falls dramatically below the nominal value, there is a significant effect on muon transmission.

Frequency [MHz]	Total voltage [MV]	Number of cavities	Peak gradient [MV/m]	Steady-state power (per cavity) [MW]	Peak RF Power (per cavity) [MW]
Rotator					
230.19	18	3	12	1.68	2.3
226.13	18	3	12	1.71	2.3
222.59	18	3	12	1.74	2.3
219.48	18	3	12	1.76	2.4
216.76	18	3	12	1.78	2.4
214.37	18	3	12	1.8	2.4
212.48	18	3	12	1.82	2.4
210.46	18	3	12	1.84	2.5
208.64	24	4	12	1.85	2.5
206.9	24	4	12	1.86	2.5
205.49	24	4	12	1.88	2.5
204.25	30	5	12	1.9	2.6
203.26	30	5	12	1.91	2.6
202.63	30	5	12	1.92	2.6
202.33	30	5	12	1.92	2.6
Total	336	56			140
Cooler					
201.25	750	100	15	3	4

TABLE III. Front end RF requirements for the rotator and cooler systems.

9. Magnet Requirements and Design

The front end requires a tapered solenoid that takes the magnetic field from 20 T at the production target to 1.5 T in 15 m. Here we describe solenoids in the drift, buncher and rotator that produce a 1.5 T constant magnetic field and solenoids in the cooler that produce the alternating solenoid field configuration. These coils are summarized in Table V.

The 1.5 T solenoids must accommodate the beam pipe, with a 30 cm radius. Within the buncher and rotator, they must also accommodate RF cavities with outer radii of 60 cm. This can be achieved using coils with an inner radius of 68 cm and a conductor radial thickness of 4 cm, so that the cavities fit entirely within the coils. A length of 50 cm spaced at 75 cm intervals leaves a gap of 25 cm between coils, matching the periodicity of the cooling channel and enabling access for room temperature services such as vacuum and RF power feeds. The required current for these coils is 47.5 A/mm^2 to give a total current of 0.95 MA-turns. The coils are therefore large enough to accommodate the beam pipe, RF and diagnostics, and added shielding. A smaller radius could be used in the first 60 m, which has no RF. The 135 m transport requires 180 such magnets.

The cooling system requires strong alternating sign coils that are placed between RF cavities, fitting within the 25 cm inter-cavity spaces (Fig. 4). The coils are 15 cm long with inner radius 35 cm, radial thickness 15 cm and current density of $\pm$107 A/mm^2 to give a total current of 2.4 MA-turns. The coil currents alternate in direction from coil to coil. These coils produce an on-axis solenoid field that

Z Position [m]	Phase [°]	Peak gradient [MV/m]	Frequency [MHz]	Length [m]	Number	Z-Separation [m]
81.48	0.0	3.42	319.63	0.40	1	0.0
85.03	0.0	4.894	305.56	0.40	2	0.40
88.405	0.0	4.17	293.93	0.40	2	0.40
91.01	0.0	5.34	285.46	0.45	2	0.45
93.26	0.0	6.36	278.59	0.45	2	0.45
95.285	0.0	4.94	272.05	0.45	3	0.45
97.535	0.0	5.61	265.8	0.45	3	0.45
99.785	0.0	6.3	259.83	0.45	3	0.45
102.03	0.0	6.97	254.13	0.45	3	0.45
104.28	0.0	7.65	248.67	0.45	3	0.45
106.53	0.0	8.31	243.44	0.45	3	0.45
108.78	0.0	9.01	238.42	0.45	3	0.45
111.03	0.0	9.71	233.61	0.45	3	0.45
112.98	5.0	13.0	230.19	0.5	3	0.75
115.23	5.0	13.0	226.13	0.5	3	0.75
117.48	5.0	13.0	222.59	0.5	3	0.75
119.73	5.0	13.0	219.48	0.5	3	0.75
121.98	5.0	13.0	216.76	0.5	3	0.75
124.23	5.0	13.0	214.37	0.5	3	0.75
126.48	5.0	13.0	212.28	0.5	3	0.75
128.73	5.0	13.0	210.46	0.5	3	0.75
130.98	5.0	13.0	208.64	0.5	4	0.75
133.98	5.0	13.0	206.9	0.5	4	0.75
136.98	5.0	13.0	205.49	0.5	4	0.75
139.98	5.0	13.0	204.25	0.5	5	0.75
143.73	5.0	13.0	203.26	0.5	5	0.75
147.48	5.0	13.0	202.63	0.5	5	0.75
151.23	5.0	13.0	202.33	0.5	6	0.75
155.1	35.0	16.0	201.25	0.5	100	0.75

TABLE IV. Full list of RF cavities. Cavities are grouped by frequency. Position is the position of the upstream edge of the first cavity in the group. Z-Separation is the distance between the centers of each cavity in the group. 0° is bunching mode while 5° and 35° are partially accelerating modes.

	Length [m]	Inner radius [m]	Radial thickness [m]	Current density [A/mm^2]	Number required
Initial transport	0.5	0.68	0.04	47.5	180
Cooling channel	0.15	0.35	0.15	±107	100

TABLE V. Summary of front end magnet requirements.

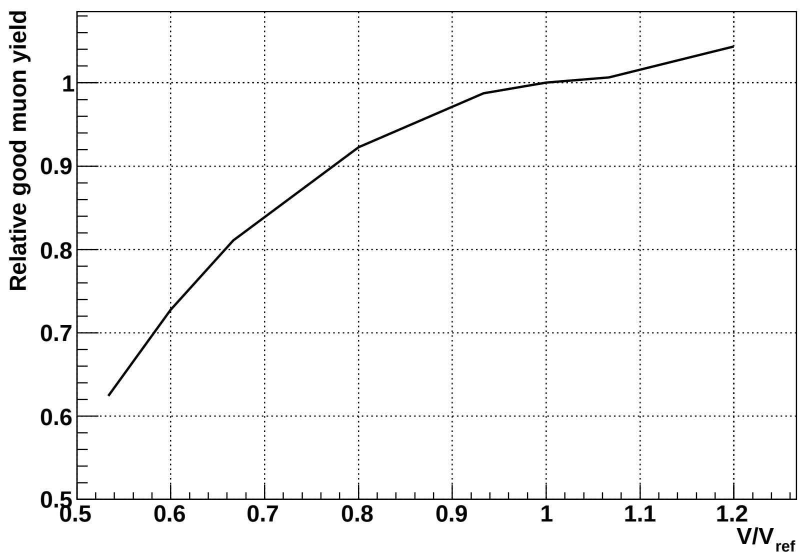

FIG. 5. Fractional change of particles within the nominal acceptance as a function of the peak field gradient, simulated in ICOOL. V_{ref} is the baseline gradient; all cavities were scaled by the constant V/V_{ref}.

varies from +2.8 T to −2.8 T over a 1.5 m period, following an approximately sinusoidal dependence. Maximum fields in the cooling cell volume are 5 T near the coil surfaces. 100 such coils are needed in a 75 m cooling system.

10. Beam Losses

There are significant particle losses along the beam line and these may result in a large energy deposition in superconducting magnets and other equipment. Two main risks have been identified: energy deposition by all particles may cause superconducting equipment to quench; and energy deposition by hadrons and other particles may activate equipment preventing handling for maintenance.

In Fig. 6, the power deposited by transmission losses per unit length from various particle species is shown as a function of distance along the channel. Note that energy deposition in RF windows and absorbers is not included in this calculation. It is expected that this equipment will absorb several kilowatts of beam power from each particle species.

For hands-on maintenance of equipment, 1 W/m is considered a maximum beam loss from protons, while magnets are expected to quench with beam losses above a few tens of W/cm^3. Several schemes are envisaged to control the beam losses in order to reduce energy depositions below these values.

Three devices are under study for reducing the transmission losses in the front end.

- Low momentum protons may be removed using a proton absorber. This device takes advantage of the different stopping distance of protons compared with other particles in material.
- Particles with high momenta outside of the acceptance of the front end may be removed using a pair of double chicanes. Dispersion is induced in the beam by means of bending magnets in

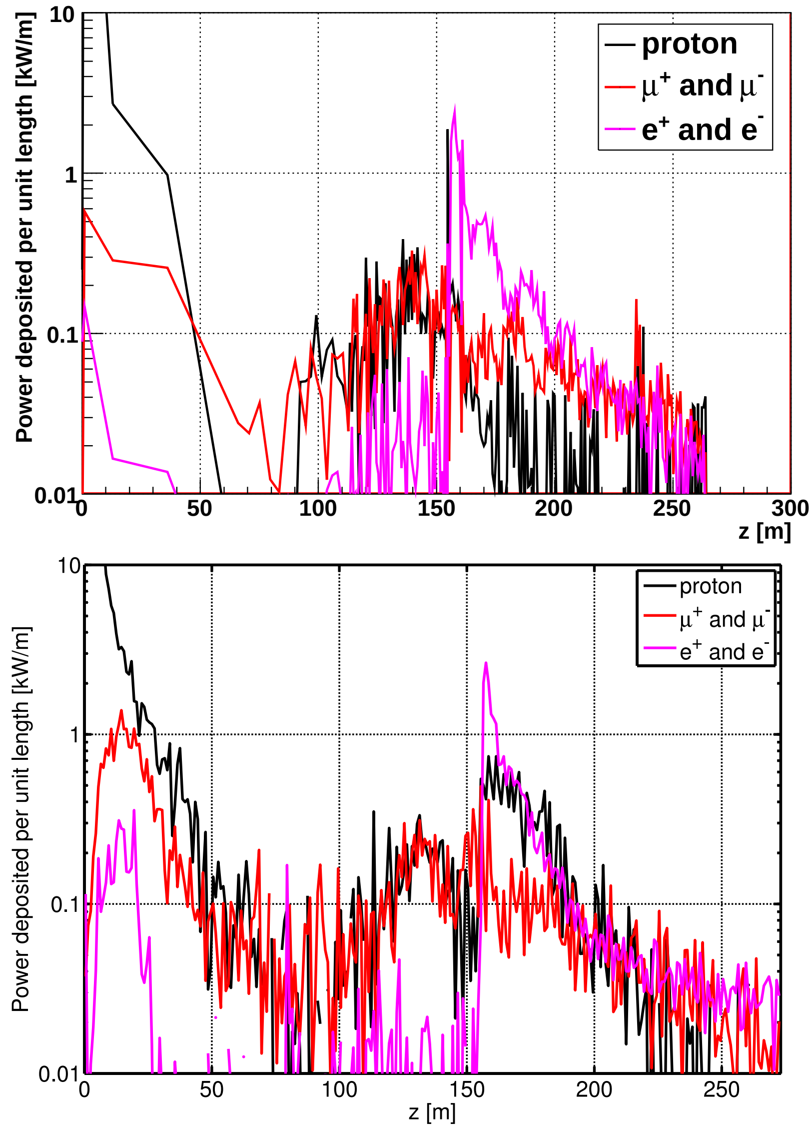

FIG. 6. Power deposited by transmission losses of various particle species in the surrounding equipment simulated in (top) ICOOL and (bottom) G4Beamline.

a chicane arrangement and high momentum particles are passed onto a beam dump. A further chicane is required to return the particles to the axis. In order to retain both muon species, one of these double chicanes is required for each sign.

- Particles with transverse amplitude outside of the acceptance of the front end may be removed using transverse collimators.

11. Summary

The Muon Collider/Neutrino Factory muon front end captures a substantial proportion of the muons produced by the target. Longitudinal capture is achieved using a buncher and energy-time phase rotation system while transverse capture is achieved using a high-field solenoid adiabatically tapered to 1.5 T and enhanced by ionization cooling. The requirement for high RF peak fields in intense magnetic fields and irradiation of the accelerator hardware due to uncontrolled losses present technical risks to the front end, but strategies are under consideration to mitigate these risks. The muon front end increases the capture rate of muons in the nominal accelerator acceptance by a factor 10.

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[1] C. Albright *et al.*, “Neutrino factory and beta beam experiments and development,” in *Joint Study on the Future of Neutrino Physics: The Neutrino Matrix*. APS, 2004. BNL-72369-2004, FNAL-TM-2259, LBNL-55478, `http://www.aps.org/policy/reports/multidivisional/neutrino/`.

[2] The ISS Accelerator Working Group *et al.*, “Accelerator design concept for future neutrino factories,” *JINST* **4** (2009) P07001.

[3] R. C. Fernow, “Recent developments on the muon-facility design-code ICOOL,” in *Proceedings of 2005 Particle Accelerator Conference, Knoxville, Tennessee*, pp. 2651–2653. IEEE, Piscataway, NJ, 2005.

[4] T. J. Roberts *et al.*, “G4Beamline particle tracking in matter-dominated beam lines,” in *Proceedings of EPAC08, Genoa, Italy*, EPAC. European Physical Society Accelerator Group, 2008.

[5] R. C. Fernow, “Physics analysis performed by ECALC9,” Tech. Rep. NFMCC-doc-280, Neutrino Factory/Muon Collider, 2003. `http://nfmcc-docdb.fnal.gov/`.

[6] G. Penn and J. S. Wurtele, “Beam envelope equations for cooling of muons in solenoid fields,” *Phys. Rev. Lett.* **85** (2000) 764.

[7] S. Agostinelli *et al.*, “GEANT4—a simulation toolkit,” *Nucl. Instrum. Methods A* **506** (2003) 250–303.

[8] R. C. Fernow, “Modeling solenoids using coil, sheet and block conductors,” Tech. Rep. NFMCC-doc-281, Neutrino Factory/Muon Collider, 2003. `http://nfmcc-docdb.fnal.gov/`.

[9] C. T. Rogers, “Modelling of the neutrino factory muon front end in ICOOL 3.10 and 3.20,” Tech. Rep. UK Neutrino Factory Note 57, UK Neutrino Factory, 2010. `http://hepunx.rl.ac.uk/uknf/wp1/`.

[10] D. Huang *et al.*, “RF studies at fermilab MuCool test area,” in *Proceedings of the 23rd Particle Accelerator Conference*. 2010.

[11] A. Moretti *et al.*, “Effects of high solenoidal magnetic fields on rf accelerating cavities,” *Phys. Rev. ST Accel. Beams* **8** (2005) 072001.

[12] R. B. Palmer *et al.*, “RF breakdown with external magnetic fields in 201 and 805 MHz cavities,” *Phys. Rev. ST Accel. Beams* **12** (2009) 031002.