Charge Recombination in the Muon Collider Cooling Channel

R. C. Fernow and R. B. Palmer

Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA

Abstract. The final stage of the ionization cooling channel for the muon collider must transversely recombine the positively and negatively charged bunches into a single beam before the muons can be accelerated. It is particularly important to minimize any emittance growth in this system since no further cooling takes place before the bunches are collided. We have found that emittance growth could be minimized by using symmetric pairs of bent solenoids and careful matching. We show that a practical design can be found that has transmission ~99%, emittance growth less than 0.1%, and minimal dispersion in the recombined bunches.

Keywords: muon collider, cooling channel, bent solenoid.
PACS: 29.27.Eg, 41.75.Lx, 41.85.Ja

INTRODUCTION

The cooling channel is a crucial part of any future muon collider. Achieving luminosities on the order of ~10^{34} cm^{-2} s^{-1} requires reduction in the 6-dimensional phase space of the muon beam by a factor ~10^6 [1]. A scheme, shown in Fig. 1, has been proposed that obtains this emittance reduction by using a series of ionization cooling and auxiliary stages [2]. The 6D cooling takes place in helical channels that can only transport muons of a given sign. In addition, the final stage of transverse cooling makes use of induction linacs, which also naturally prefer to work with particles of a given sign. For these reasons, the cooling channel begins with a charge separation stage. A preliminary design of this system made use of bent solenoids [3]. At the end of the cooling channel, the charged bunches need to be recombined transversely into a single channel before acceleration takes place to higher energies. The charge recombination system, described in this paper, also makes use of bent solenoids. It is particularly important that this process take place with minimal emittance growth since no further cooling is available before the bunches reach the collider ring. This is the only part of this proposed cooling channel that has not previously been simulated at some level.

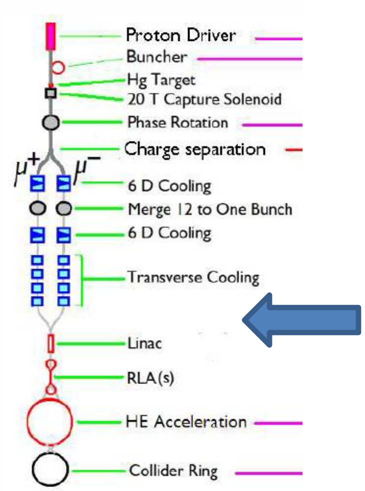

FIGURE 1. Layout of a potential muon collider; the cooling channel begins with Charge Separation and ends at the large arrow where Charge Recombination takes place.

MOTION IN A BENT SOLENOID

Since the cooling channel uses solenoidal focusing, it is natural to do charge recombination using bent solenoids. A bent solenoid has dispersion in the plane perpendicular to the plane of bending. Consider an orthogonal coordinate system with one axis along a reference trajectory (**s**), one axis perpendicular to the bending plane (**y**), and the third axis (**x**) in the x-s plane. To lowest order, the equations of motion for the transverse coordinates for the case of piecewise-constant solenoid and dipole fields are given by

$$x'' = h + \frac{q}{p_s}(y' b_s - b_0)$$
$$y'' = -\frac{q}{p_s} x' b_s$$

where **h** is the curvature, **q** is the charge, $\mathbf{p_s}$ is the component of momentum along the reference trajectory, $\mathbf{b_s}$ is the solenoid field, and $\mathbf{b_0}$ is the dipole field. For the case where a particle starts on-axis and is directed along the reference trajectory, the solution of these coupled differential equations is

$$x(s) = \frac{A}{\alpha b_s} \cos\frac{2\pi}{\lambda_L} s + \frac{A}{\alpha b_s}$$

$$\boldsymbol{y(s) = \frac{A}{\alpha b_s} \sin\frac{2\pi}{\lambda_L} s - A\, s}$$

where

$$\alpha = \frac{q}{p_s}$$

$$A = \frac{(h - \alpha b_0)}{\alpha b_s}$$

and the Larmor wavelength is

$$\lambda_L = \frac{2\pi p_s}{q b_s}$$

We see that the **x** motion is oscillatory with a period given by the Larmor wavelength. The amplitude of the oscillations can be reduced by increasing the solenoid field or can be zeroed out entirely by using a superimposed dipole field with the magnitude

$$\mathrm{b_0} = \frac{\mathrm{h\ p_s}}{\mathrm{q}}$$

The **y** motion contains an oscillatory term that is 90^o out of phase with the **x** motion. However, the important feature here is that the **y** motion also contains a term that is linear in **s**. Written out, the deflection in **y** is given by

$$\Delta \mathrm{y} = \frac{\mathrm{p_s}\left(\mathrm{h} - \frac{\mathrm{q b_0}}{\mathrm{p_s}}\right)}{\mathrm{q b_s}}\ s$$

The amount of deflection increases for increasing $\mathbf{p_s}$, **h**, and **s**, and decreases for increasing $\mathbf{b_s}$. The deflection has the opposite sign as the charge, and can be reduced to zero by choosing a superimposed dipole field, as above. For a beam of particles, a vertical dispersion

$$D = -\frac{h\, p_{ref}\, s}{q b_s}$$

is present, regardless of whether the mean deflection or the dipole field is zero.

These expressions have been checked using the tracking code ICOOL [4], which uses the full equations of motion. Figure 2 (left) shows the **x** and **y** trajectories for a positive particle through a bent solenoid two Larmor wavelengths long and with no dipole field. Figure 2 (right) shows the same channel for a beam with a superimposed dipole field. Note that although the mean deflection is zero, there is vertical dispersion in the beam after passing through the bent solenoid.

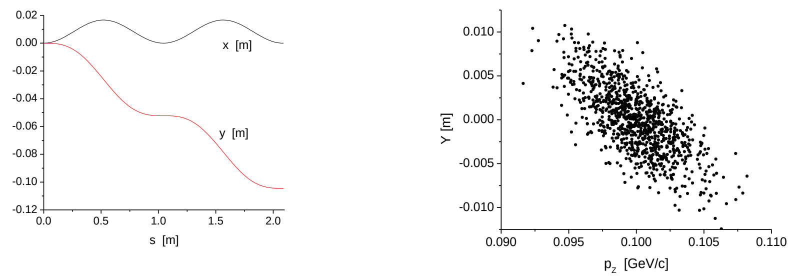

FIGURE 2. Tracking results for p = 100 MeV/c, b_s = 2 T, h = 0.3 m^{-1}. Single particle results (left) have $b_0 = 0$; beam results for the same channel (right) have b_0 = 0.1 T.

In cases where the dipole field is zero, careful matching is necessary to avoid emittance growth. A method to do this has been given by Norem [5]. Assume we have a bent solenoid of arbitrary length with a constant curvature **h**. In this method, the bent solenoid is matched to straight solenoid channels with a matching section $\lambda_L/2$ long with curvature **h**/2. Simulations with ICOOL have confirmed that single particle tracking using this method of matching does in fact remove the oscillations on the trajectories.

DESIGN STRATEGY

Figure 3 shows a schematic of the proposed system for charge recombination.

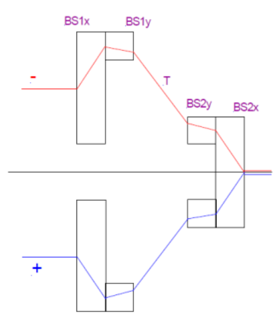

FIGURE 3. Vertical slice of the charge recombination channel. BS1x and BS2x are bent solenoids that bend in the horizontal plane; BS1y and BS2y are bent solenoids that bend in the vertical plane; T is a straight solenoid transport pipe.

The preliminary design was done with ICOOL. Practical issues concerning the location and current densities in the coils was done with the program G4beamline [6]. We always use pairs of bent solenoids with opposite curvature in a given plane in order to remove the dispersion. We use superimposed dipole fields with BS1x, BS1y and BS2y to keep the beam centered on the reference trajectory. Norem matching is used entering and leaving BS2x, where the actual transverse charge recombination takes place. BS1y and BS2y were introduced to help give adequate separation of the channels coming into BS2x and to avoid the problem of overlapping coils. We don't put any RF cavities in this channel to avoid disrupting the dispersion cancellation. Note that this whole system could be rotated by 90^o around the outgoing beam axis if there were practical advantages in doing so.

During the preliminary design work on this channel, we found that the emittance growth was independent of the ratio p / b_s over a large range, as shown in Fig. 4. Note that the emittance growth is minimal for solenoid field strengths between 2 and 8 T.

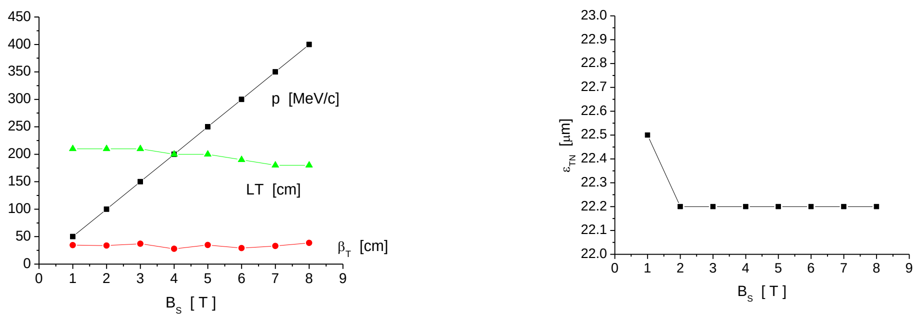

FIGURE 4. Scaling of the emittance growth as a function of p / b_s. (left) channel parameters, L_T is the length of the transport solenoid, β_T is the beta function of the incoming beam; (right) simulated transverse emittance growth.

CHARGE RECOMBINATION SYSTEM

The charge recombination system uses the positive and negative beams coming out of the final transverse cooling channel. We assume that a short section of accelerator is added after each final cooling channel to raise the momentum of the beams from ~40 MeV/c to 100 MeV/c. Table 1 gives the incoming beam parameters.

TABLE 1. Incoming beam parameters.

Quantity	Value	Units
ε_{TN}	22	µm
ε_{LN}	72	mm
p	100	MeV/c
$\sigma_x = \sigma_y$	2.8	mm
$\sigma_{px} = \sigma_{py}$	0.83	MeV/c
β_T	34	cm
σ_z	300	cm
σ_{pz}	2.47	MeV/c

We used a uniform solenoid field of 2 T throughout the whole channel. A vertical dipole field of 0.1 T and curvature 0.3 m^{-1} was used in BS1x. BS1y and BS2y used horizontal dipole fields of ±0.023 T and curvatures ±0.07 m^{-1}. BS2x did not have a dipole field. The two beams were separated by ±5.1 cm at the entrance toBS2x.

The two beams on the entrance face to BS2x are shown on the left side in Fig. 5 and the combined beams at the exit face are shown on the right.

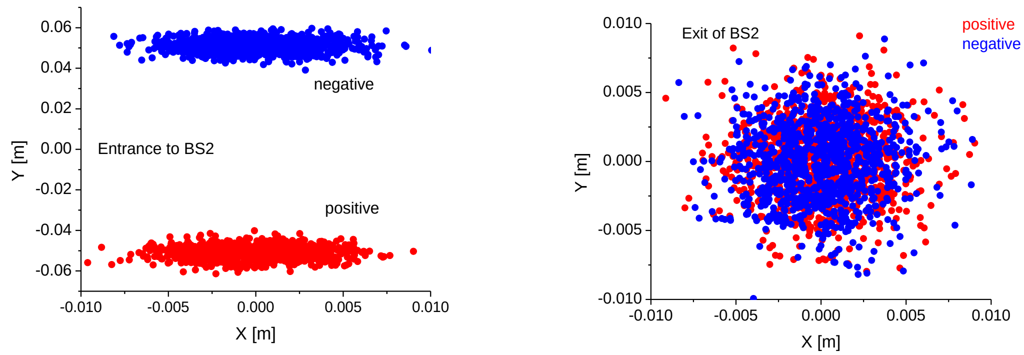

FIGURE 5. (left) Positive and negative beams at the entrance face to BS2x; (right) combined beams at the exit face of BS2x.

We see that the two beams are combined symmetrically in the transverse plane. However, the incoming path lengths must be adjusted so that the bunches are still separated longitudinally by ½ of the RF wavelength of the accelerator channel that follows the charge recombination system. Figure 6 shows the dispersions at the end of the channel for the positive beam.

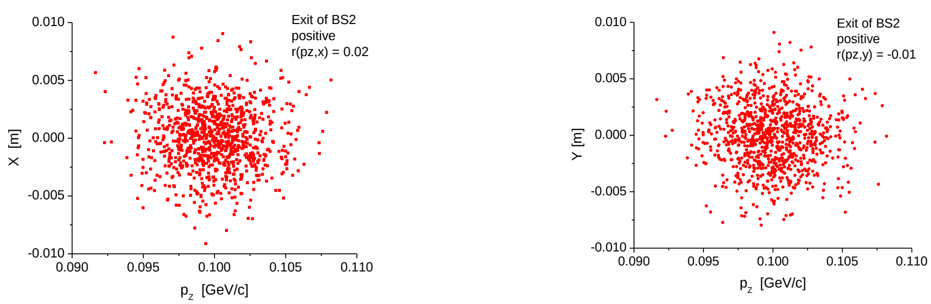

FIGURE 6. Dispersions at the end of the channel for positive particles; (left) dispersion in x, (right) dispersion in y.

The dispersion is almost completely removed in the final beam. The results for the negative beam are similar. Figure 7 shows the statistical transverse emittance as a function of distance along the channel.

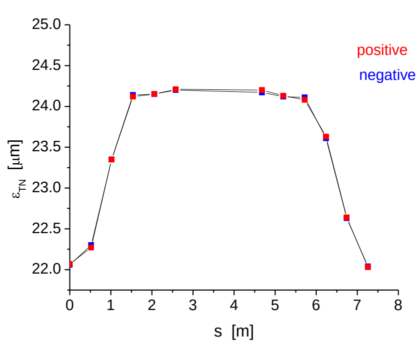

FIGURE 7. Normalized transverse emittance as a function of distance along the charge recombination channel.

The apparent increase in emittance in the interior of the channel is due to the presence of dispersion in the beam. We see that the final emittance, computed at the end after the dispersion has been removed, is very close to the initial value. The transverse emittance growth is less than 0.1%, as is the longitudinal emittance growth. The transmission through the channel, including muon decays, is greater than 99%.

A G4beamline model of the channel is shown in Fig. 8.

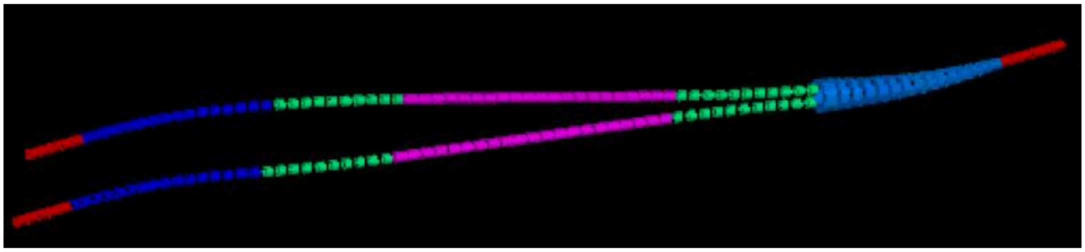

FIGURE 8. G4beamline model of the charge recombination channel; external beamlines (red), BS1x (dark blue), BS1y and BS2y (green), straight transport (magenta), BS2x (light blue).

The coil dimensions were adjusted to avoid geometric overlaps and to limit the engineering current densities to below 105 A / mm^2. The coils in BS2x were tapered in radius to avoid any flux leakage at the exit face.

This design should enable the cooled muon bunches in the muon collider cooling channel to be recombined transversely. Simulations with the simplified field model used here showed that the recombination takes place efficiently and has minimal emittance growth. The actual channel will be made of discrete coils, so further studies should be made to find the variations in the magnetic field along the reference trajectory, and to determine whether this has a significant effect on the emittance growth.

ACKNOWLEDGMENTS

This manuscript has been authored by employees of Brookhaven Science Associates, LLC under Contract No. DE-AC02-98CH10886 with the U.S. Department of Energy. The publisher by accepting the manuscript for publication acknowledges that the United States Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce the published form of this manuscript, or allow others to do so, for United States Government purposes.

REFERENCES

1. R. B. Palmer and R. C. Fernow, “An overview of muon colliders”, *ICFA Beam Dynamics Newsletter*, August 2011, pp. 22-54.
2. R. B. Palmer, “Muon collider progress”, *Proc. 2009 Part. Accel. Conf*., pp. 652-656.
3. R. B. Palmer and R. C. Fernow, “Charge separation for muon collider cooling”, *Proc. 2011 Part. Accel Conf.*, pp. 103-105.
4. R. C. Fernow, “Recent developments on the muon-facility design-code ICOOL”, *Proc. 2005 Part. Accel. Conf*., pp. 2651-2653.
5. J. Norem, “Coupling sections, emittance growth, and drift compensation in the use of bent solenoids as beam transport elements”, *Phys. Rev. Spec. Top.-Accel. Beams* **2**, 054001 (1999)
6. T. J. Roberts *et al*, "G4beamline particle tracking in matter-dominated beamlines", *Proc. 2011 Part. Accel Conf.*, pp. 373-375.