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Hybrid Arc Cell Studies: Status Report

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Abstract

I report on the status, at the end of FY12, of the studies of an arc cell for a hybrid synchrotron accelerating from 375 GeV/c to 750 GeV/c in momentum. Garren produced a complete lattice that gives a good outline of the structure of a hybrid synchrotron lattice. It is, however, lacking in some details: it does not maintain a constant time of flight, it lacks chromaticity correction, its cell structure is not ideal for removing aberrations from chromaticity correction, and it probably needs more space between magnets. I have begun studying cell structures for the arc cells to optimize the lattice performance and cost. I present some preliminary results for two magnets per half cell. I then discuss difficulties encountered, some preliminary attempts at resolving them, and the future plans for this work.

1. Introduction

A hybrid synchrotron is a concept of Don Summers that uses interleaved superconducting and ramped dipoles to achieve a high average bending field simultaneously with rapid acceleration. This allows the efficient acceleration of muons to high energy. A more complete introduction with references is available in [1].

In [1] and [2], Garren and I present a first-pass design that uses 8 superperiods, each with 6 arc cells, 3 cell straights, and dispersion suppressors that eliminate both dispersion and closed orbit motion with energy in the straights. The dispersion suppressors maintain a high average bend field, similar to that in the arcs. I will refer to this as the Garren design.

The Garren design gives a good picture of what the structure of the machine should be. The dispersion suppressors maintaining a high average bend field is a particular advantage. However, there are still some issues to be addressed in this lattice:

- The time of flight of the reference orbit must be independent of the reference momentum (this does not mean that the frequency slip factor is zero).
- Ideally the arcs should have an integer tune so that identical sextupoles can be used at all points in the arc to provide chromaticity correction while eliminating nonlinear aberrations at low orders. While this is not strictly necessary, it will probably work better than other solutions.
- The existing drift lengths are probably too short. Values in the range of 0.5 m [3] to to 2.0 m [4] have been suggested to me; I will optimistically take the lower value.
- 6 superperiods are likely to be sufficient.
- The lattice needs a tuning mechanism, which will likely be in the straight.
- We would like to reduce the required aperture, which in the Garren design is coming about equally from dispersion side and orbit motion.
- The existing dipole structure within an arc cell is somewhat arbitrary; it should be optimized.

2. Arc Cell Optimization

The goal of the initial study is to look at different configurations of warm and cold dipoles in an arc cell to determine the optimum configuration. The arc cells will have time of flight and tune independent of momentum. The quadrupoles will be split in half with some drift space allocated for sextupoles, but no sextupoles will be used in the initial study. I will need to revisit the study when sextupoles are included since there will probably be closed orbit motion with momentum in the sextupoles.

For this study I make the following assumptions:

- The lattice cell is FODO.
  - The dipoles are split in half, with two inter-magnet drifts between them. Eventually a sextupole will go in this space (I will later add a third drift worth of placeholder for the sextupole).
  - Reflection symmetry about either quadrupole will be maintained.
- Superconducting dipoles have a 8.0 T field.
- Warm dipoles have a maximum field of 1.8 T.
- Quadrupoles have a maximum field of 1.5 T, and the ratio of the pole tip radius to the maximum beam radius is 1.3.

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- There are 8 superperiods (later I will modify this to 6).
- The arc in each superperiod will have an integer tune to facilitate correction of low-order nonlinear terms from the chromaticity correction sextupoles.
- There is 75 cm of space between magnets (later I will modify this to 50 cm).
- The normalized beam emittance is 25 µm, and the energy spread is 750 MeV.

An initial guess for dipole lengths can be obtained based on the minimum and maximum momenta and desired bend angles per cell:

$$L_W = \frac{\theta}{2B_W c}\left(\frac{p_+c}{e} - \frac{p_-c}{e}\right) \tag{1}$$

$$L_C = \frac{\theta}{2B_C c}\left(\frac{p_+c}{e} + \frac{p_-c}{e}\right) \tag{2}$$

where L_W is the total length of warm dipole in the cell, L_C is the total length of cold dipole in the cell, B_W is the warm dipole field, B_C is the cold dipole field, p_- is the minimum momentum, and p_+ is the maximum momentum. These lengths will be treated as a parameter for optimization.

The tunes are chosen to be k_x/N and k_y/N in the horizontal and vertical planes, respectively, where $0 < k_\alpha < N/2$, and N is the number of cells per superperiod.

2.1. *Two Dipoles per Half Cell*

The configuration of two dipoles per half arc cell is not meant to represent a desired configuration; it is merely the first step in studying the properties of the hybrid arc cell lattices. We will learn important properties of the arc cell from these studies, but the values (apertures and circumferences) will be significantly larger than they would be in a real machine.

There are two possible arrangements: the cold dipole can be near either the F or D quadrupole. In optimizing this lattice, find closed orbits at 375 GeV/c and 750 GeV/c with the warm dipoles at +1.8 T and −1.8 T, respectively. The following parameters are varied:

- The quadrupole fields at the maximum and minimum momentum
- The quadrupole lengths
- The warm and cold dipole lengths

These are adjusted to meet the following criteria:

- At both momenta, the tunes are the desired values.
- The maximum field seen by the beam (including closed orbit motion, dispersion size, and emittance size) is set equal to (1.5/1.3) T.
- The time of flight at the two momenta is the same.

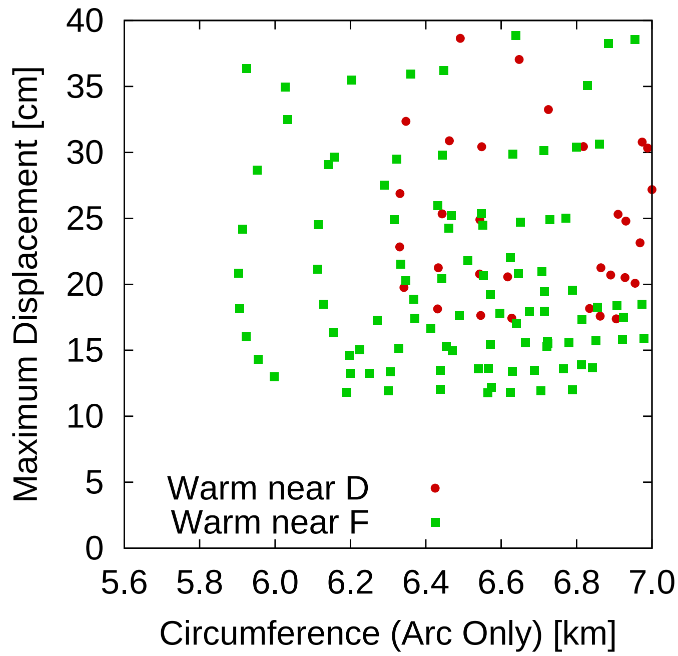

Figure 1: Maximum horizontal displacement in an arc cell as a function of ring circumference.

- Make the sum over the two quadrupoles of the ratio of the average of the minimum and maximum beam horizontal position to the difference between the maximum and minimum positions. This is equivalent to $x_{F,\min}x_{D,\min} = x_{F,\max}x_{D,\max}$. This is an attempt to minimize the beam excursion while taking into account the significantly larger beam excursion in the F quadrupole. The positions take into account the closed orbit position, the dispersion size, and the emittance size.

The results of the optimizations are shown in Figs. 1–3. Figure 1 is the key plot: it demonstrates that having the warm dipole adjacent to the focusing quadrupole gives the smallest horizontal aperture for a given circumference. The beam tends to move less in the fixed field dipole and more in the warm dipole, and so when the warm dipole is near the focusing quadrupole, the focusing quadrupole works with the warm magnet to bend the beam back in at higher energies, whereas when it is near the defocusing quadrupole, the defocusing quadrupole is working against the beam motion. [!t]

Figure 2 shows that even shorter circumferences can be achieved with fewer arc cells per superperiod. However, Fig. 3 shows that in this case, the horizontal aperture will be much larger (in fact, it is off scale both in this graph and in Fig. 1). While the runs were not yet extended out to more arc cells per superperiod, it seems likely that while lower apertures could be achieved, this would be at the cost of increased circumference. While the quadrupoles would shorten as the aperture decreased, the increase in the number of drifts causes the circumference to increase.

Figure 3 also demonstrates that for a given number of cells, having the warm magnet near the D will give a lower maximum displacement; this is because the motion in the warm magnet

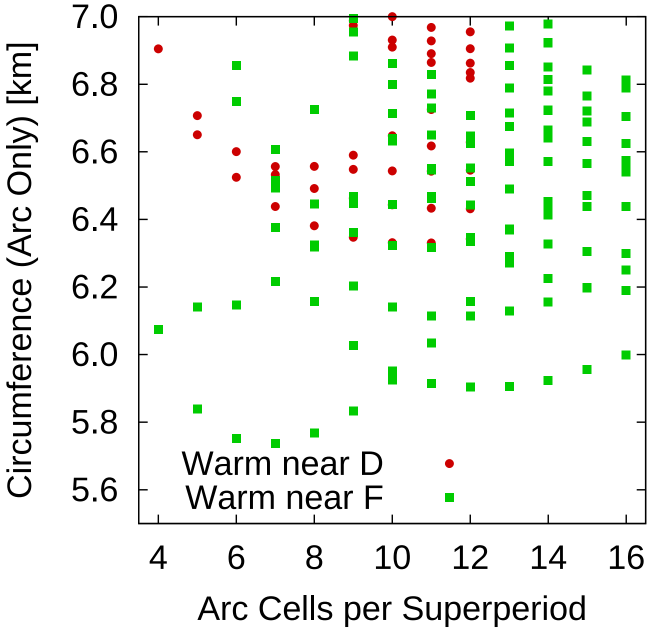

Figure 2: Ring circumference, ignoring straights and matching sections, for two dipoles per half cell as a function of the number of cells per superperiod. Multiple points for each value a given number of cells are for different horizontal and vertical tunes.

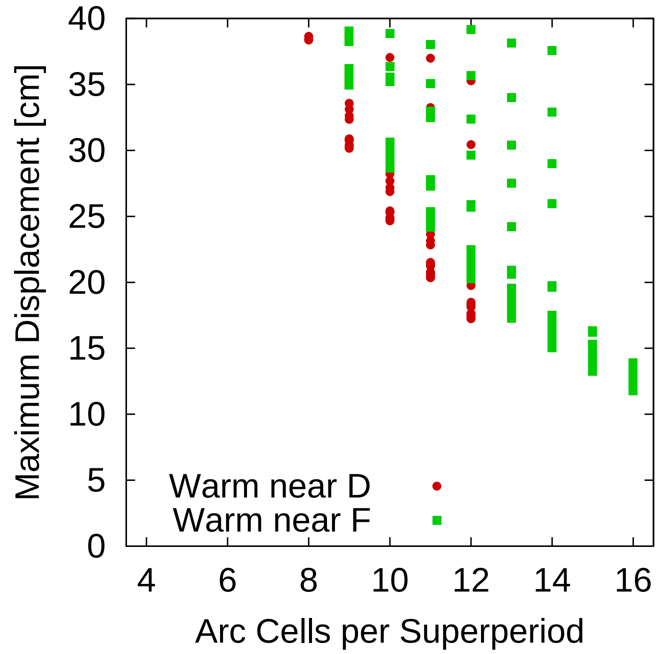

Figure 3: Maximum horizontal displacement (includes closed orbit motion, dispersion size, and emittance size) in an arc cell as a function of the number of cells per superperiod.

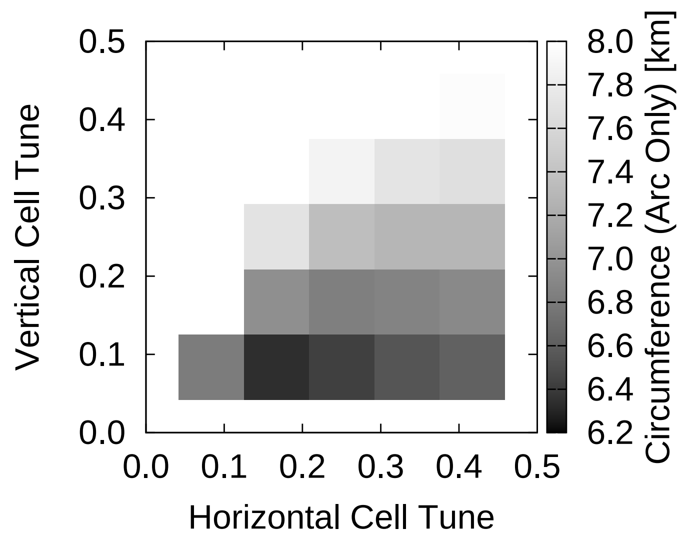

Figure 4: Circumference as a function of tune, for 12 arc cells per superperiod. Warm dipole is near the defocusing quadrupole.

opposing the bend direction in the defocusing quadrupole tends to reduce the overall displacements, but at the cost of longer quadrupoles. The circumference is thus increased, and thus as shown in Fig. 1, the results displacement for a given circumference is better when the warm magnet is near the focusing quadrupole.

Figures 4–7 show the dependence of the circumference and horizontal aperture on the tune. The circumference prefers a low vertical tune and a horizontal tune somewhat below 0.25. This is because reducing vertical focusing reduces the required integrated quadrupole strengths while having no significant impact on the horizontal motion (note that the beam size is dominated by the dispersion size [2], so vertical aperture is not a significant problem, though maybe this should be revisited when very small vertical tunes are considered). For horizontal aperture, dependence on vertical tune is weak (as expected), but the optimal horizontal tune depends on which configuration is under consideration. For the warm magnet near the focusing quadrupole, a high horizontal tune is preferred (conflicting with the desired tune for reducing the circumference), while for the warm magnet near the focusing quadrupole, a horizontal tune near 0.25 is preferred (similar to the requirement for reducing circumference).

I expect that these results will continue to hold up for an even number of dipoles per half cell: namely that the dipole nearest the focusing quadrupole should be warm, and that a low vertical tune and a horizontal tune at or just below 0.25 is preferred. We expect significantly reduced horizontal displacements and somewhat reduced circumferences with more dipoles per half cell, up to a the point where the increased number of intermagnet drifts begins to increase the arc cell lengths to the point where the benefits to circumference and horizontal aperture no longer appear.

A question still to be addressed is the preferred configuration with an odd number of dipoles per half cell (as in the Garren lattice). Garren demonstrated a configuration with no closed orbit motion in the quadrupoles, thus giving relatively small aperture quadrupoles. When including time of flight correction, we will need to determine whether it is more optimal to use motion in the quadrupoles to help with that correction (thus having warm magnets near the quadrupole), or whether is better to just use steering with the dipoles to accomplish this (this having cold

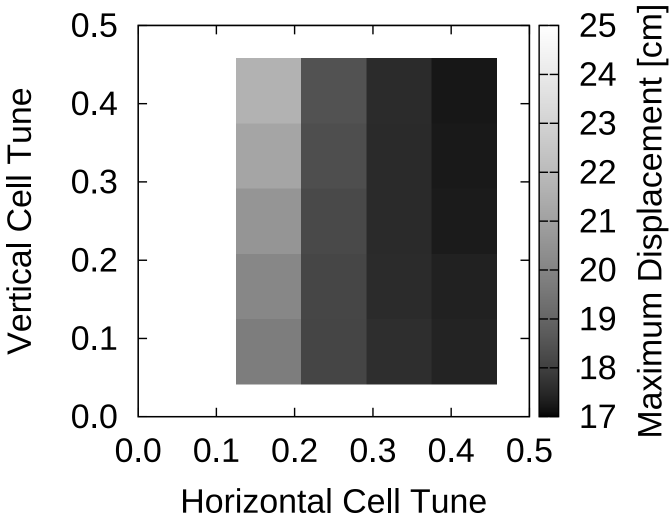

Figure 5: Maximum horizontal displacement as a function of tune, for 12 arc cells per superperiod. Warm dipole is near the defocusing quadrupole.

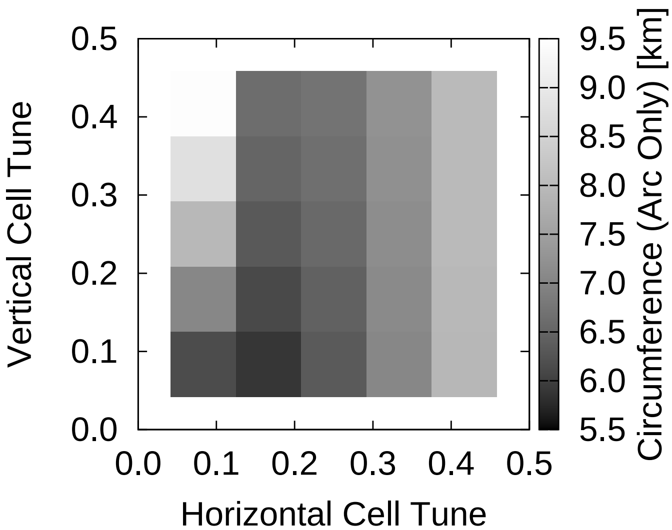

Figure 6: Circumference as a function of tune, for 12 arc cells per superperiod. Warm dipole is near the focusing quadrupole.

magnets near the quadrupoles).

2.2. Challenges and Future Steps

The biggest challenge in this study has been the automated finding of solutions for a given configuration. It appears that a solution always exists. However, analytic estimates based on a thin lens FODO lattice tend to be far off due to the split quadrupoles, the relatively long quadrupoles, and the relatively long drifts between them. Displacements and angles can then easily get large, and particles begin to go backwards when exact solutions are used.

I am addressing this in two parts: first, I am first finding a solution using paraxial approximations for everything, which has the advantage both of being fast and avoiding the issue of particles turning around. I will then start with that solution and

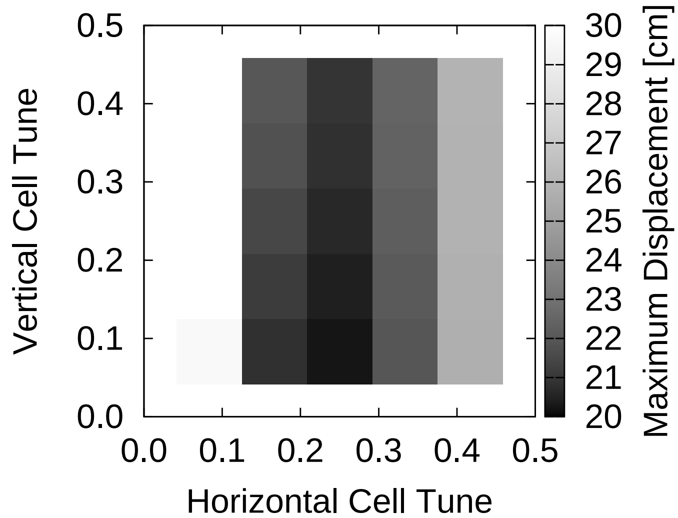

Figure 7: Maximum horizontal displacement as a function of tune, for 12 arc cells per superperiod. Warm dipole is near the focusing quadrupole.

improve it by using analytic solutions for the magnets, with exact solutions for dipoles and drifts but paraxial approximations for quadrupoles. These solutions can also be evaluated rapidly, and the improved initial guess should avoid issues of particles going backward. This solution will then be refined with exact tracking, which should converge quickly since we will be very close to the correct solution. I am augmenting this with an more robust closed orbit finder (see the appendix) which should find closed orbits as long as the initial guess does not result in a particle going backward.

Once I have a more robust optimizer working, my plans are

1. Find solutions for increasing numbers of dipoles per half cell with varying arrangements, until adding more dipoles is clearly undesirable.
2. Add chromatic correction to the preferred solution. Verify that this does not change the preferred number of dipoles and their arrangement.
3. Add dispersion suppression and straights. Adjust the time of flight correction scheme to take these additional sections into account. Again, verify that this does not change the preferred number of dipoles.
4. Verify sufficient dynamic aperture through tracking.

Appendix A. Robust Closed Orbit Finder

Begin by defining a function $\boldsymbol{f}(\boldsymbol{z})$ which returns the phase space variables after one turn if they had the initial values of $\boldsymbol{z}$. The closed orbit is the solution to the equation $\boldsymbol{f}(\boldsymbol{z}) = \boldsymbol{z}$. I solve this equation using a modified version of Newton's method:

$$\boldsymbol{z}_{n+1} = \boldsymbol{z}_n + \alpha[\boldsymbol{I} - \nabla\boldsymbol{f}(\boldsymbol{z}_n)]^{-1}[\boldsymbol{f}(\boldsymbol{z}_n) - \boldsymbol{z}_n] \quad \text{(A.1)}$$

Initially $\alpha = 1$. But if the evaluation of $\boldsymbol{f}(\boldsymbol{z}_{n+1})$ fails (because the particle goes backward), I reduce α by a factor of 2 until it succeeds. As long as the initial evaluation $\boldsymbol{f}(\boldsymbol{z}_0)$ succeeds, this method seems to be very robust for finding a solution.

Most closed orbit finders require that a vector be multiplied by $(I - M)^{-1}$, where M is a symplectic matrix. This can be computed explicitly with much less work than one would expect. A $2n \times 2n$ symplectic matrix M has a characteristic polynomial of the form [5]

$$\sum_{k=0}^{2n} c_{nk} M^k = 0 \tag{A.2}$$

with $c_{n,2n-k} = c_{nk}$, and $c_{n0} = c_{n,2n} = 1$. The first three coefficients (the only ones we need) can be expressed as [6]

$$c_{n1} = -\sum_{1 \le i \le n} M_{ii} \tag{A.3}$$

$$c_{n2} = \sum_{1 \le i < j \le n} \begin{vmatrix} M_{ii} & M_{ij} \\ M_{ji} & M_{jj} \end{vmatrix} \tag{A.4}$$

$$c_{n3} = -\sum_{1 \le i < j < k \le n} \begin{vmatrix} M_{ii} & M_{ij} & M_{ik} \\ M_{ji} & M_{jj} & M_{jk} \\ M_{ki} & M_{kj} & M_{kk} \end{vmatrix} \tag{A.5}$$

Next, compute the inverse of $I - M$. To do this, we first write the inverse in the form

$$(I - M)^{-1} = \frac{1}{2} \left[I + \sum_{k=1}^{n} \lambda_{nk} (M^k - M^{-k}) \right] \tag{A.6}$$

The inverse of powers of a symplectic matrix are trivial to compute from the corresponding power:

$$(M^{-k})_{ij} = J_{i\sigma(i)} J_{j\sigma(j)} (M^k)_{\sigma(j)\sigma(i)} \tag{A.7}$$

$$\sigma(i) = (2, 1, 4, 3, 6, 5) \tag{A.8}$$

$$J_{i\sigma(i)} = (+1, -1, +1, -1, +1, -1) \tag{A.9}$$

We then multiply this by $I - M$, use the characteristic polynomial to eliminate higher powers of M, then zero the coefficients of the remaining nonzero powers of M. The results are

$$\lambda_{nk} = \begin{cases} \dfrac{2 + 2\sum_{j=1}^{n-k-1} c_{nj} + c_{n,n-k}}{2 + 2\sum_{j=1}^{n-1} c_{nj} + c_{nn}} & k < n \\[2ex] \dfrac{1}{2 + 2\sum_{j=1}^{n-1} c_{nj} + c_{nn}} & k = n \end{cases} \tag{A.10}$$

When the orbit is known to be in the midplane (as it is for our problem), one should only compute the elements in the horizontal plane, to avoid singularities arising from the vertical plane potentially causing difficulties with the matrix inversion.

[1] A. A. Garren and J. S. Berg, "A Lattice for a Hybrid Fast-Ramping Muon Accelerator to 750 GeV," MAP-doc-4307 (2011).

[2] J. S. Berg and A. A. Garren, "Hybrid Fast-Ramping Accelerator to 750 GeV/c: Refinement and Parameters over Full Energy Range," MAP-doc-4335 (2012).

[3] H. Witte, private communication (2011).

[4] J. Tomkins, private communication (2011).

[5] A. J. Dragt, "Lie Methods for Nonlinear Dynamics with Applications to Accelerator Physics," version 13 September 2012, `http://www.physics.umd.edu/dsat/`.

[6] N. Jacobson, "Basic Algebra I," (Dover, 2009).