Beam loading and longitudinal instabilities at the Neutrino Factory and Muon Collider

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Abstract

The constituents of Muon Collider (MC) and Neutrino Factory (NF) are considered including: proton Accumulator Ring (AR), proton Compressor Ring (CR), and muon collider ring for Higgs Factory (HF). Interaction of high intensity beams with accelerating cavities is the main subject of the investigation including beam loading effects and attendant instabilities. It is shown that the beam-induced RF voltage is very large, so that feed-forward and feedback systems are needed for its compensation and for the instability suppression, whereas potential well distortion is negligible.

1 Introduction

Schematic and scenario of the Neutrino Factory and Muon Collider facilities are sketched in Fig. 1 and described below quite as they were proposed in Ref. [1]-[2].

The complex consists of proton driver, proton-to-muon converter, and Muon Collider ring and/or Neutrino Factory decay ring. The Project-X based proton driver includes 8 GeV linac, 308 m long proton Accumulator Ring, and bunch Compressor Ring of the same length. It is assumed that the linac will provide a long series of batches, 0.129 μs each, separated by

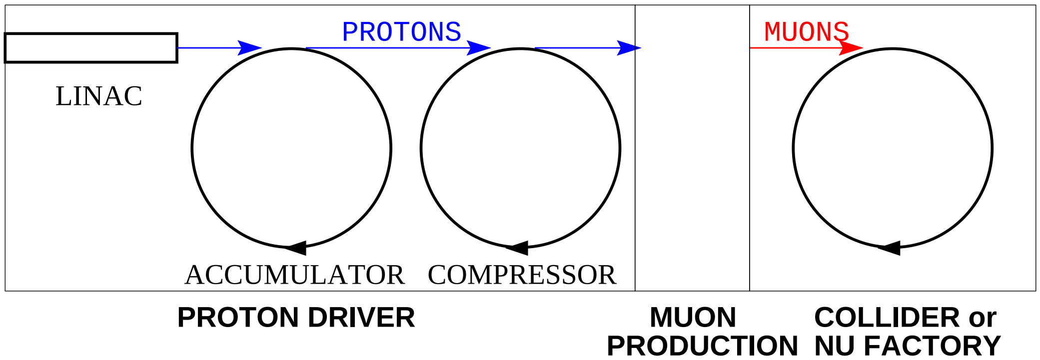


Figure 1: Schematic of Neutrino Factory or Muon Collider

intervals of the same duration. With average proton beam 5 mA, it allows to accumulate 2.1×10^{14} proton in the course of 6.73 ms that is 6510 turns of the accumulator.

The accumulated beam consists of 4 bunched which should be supported by 3.87 MHz RF system. As the accumulation is finished, all the bunches should be transfered to the CR – one of time for NF or all together for MC. There the bunches should be compressed by factor about 10 before they will be extracted and directed to the muon production facility. The last includes a production target, muon cooling systems, bunch coalescing ones, etc. These parts are not included in our exploration and therefore are not specified in the picture. Produced muon bunches are injected in the MC or NF.

Beam loading and attendant longitudinal instabilities are the main subjects of this work. The problem is important, first of all, because of very high beam current which pick value can reach about 100 A in the AR, 1000 A in the CR, and several hundreds A in the Higgs Factory ring. The situation is additionally dramatized in the HF because of very low beam momentum spread required for it.

2 Required and estimated parameters

Required accelerator and beam parameters are taken from Ref. [1]-[2] and compiled in Table 1. Rms bunch length at injection into AR is found with the assumption that each injected mini-pulse has rectangular shape of length of 38.5 m (0.129 μs). Actually, such a pulse is not fully matched with the AR separatrix so that the bunch sizes change rather rapidly even without beam loading (see below).

For an estimation of the beam loading effect, it is necessary to presume some parameters of accelerating cavities. Although it is mostly an engineering problem, known technical solutions have to be applied for a preliminary analysis. We have used information concerning the existing 2.5 MHz and 53 MHz cavities as it is presented in Ref. [3].

Accepted parameters of the cavities are listed in Table 2. It would be well to know harmonic composition of the beam current to calculate the beam induced voltage. However, detailed analysis and simulations are required for this, which work will be done later. As for now, it is quite possible to use average beam current for the first estimations, because resonant harmonics, typically, have the same order of value. (for example, they are just twice more of DC for short bunches). The estimation are presented in Table 3.

It is clear from these tables that compensation of the beam loading by feed-forward and, probably, feed-back systems is certainly required at all the stages, especially in the AR and CR.

Table 1: Accelerators and beams parameters

Parameter	Units	AR	CR	HF
Circumference	m	308	308	299
Total beam energy	GeV	8.94	8.94	62.5
Revolution frequency	MHz	0.967	0.967	1.00
Slippage factor	–	-0.063	-0.010	0.079
Beam intensity	10^{12}	210	210	2.0
Average beam current	A	32.5	32.5	0.32
Number of bunches/beam	–	4	4	1
RMS energy spread at injection	MeV	3.75	4.65	1.88
RMS bunch length at injection	cm	1110	875	5.64
RF harmonic number	–	4	4	200
RF frequency	MHz	3.87	3.87	200
RF voltage	kV	10	240	100

Table 2: Presumed cavity parameters

Parameter	Units	AR&CR	HF
Frequency	MHz	3.87	200
Quality Q	–	125	5000
Shunt impedance R	kΩ	50	250
R/Q	Ω	400	50
Maximal voltage per cavity	kV	20	100

Table 3: Estimated beam loading

Parameter	Units	AR	CR	HF
Number of cavities	–	1	12	1
Beam induced voltage	kV	1620	19440	160
Induced to nominal volt.	–	162	81	1.6 ($\mu^{\pm}$ together)

3 Accumulator Ring

The cavities parameters

$$R = 50\,\mathrm{k}\Omega, \quad Q = 125, \quad R/Q = 400\,\Omega$$

are accepted for the estimations. As it has been mentioned above, induced voltage about 1.6 MV is expected with such parameters at designed intensity $N = 2.1 \times 10^{14}$.

This circumstance absolutely excludes operations without compensation of the beam loading in advance by corresponding programming of the power supply (feed-forward). However, it turns out that this method is not fully adequate and does not allow to reach project intensity of the AR. The problems are caused by the beam instabilities which can appear in the last stage of the accumulation being initiated by accidental factors like statistical uncertainty of initial distributions, noise, etc. Some examples are presented in Fig. 2 and Fig 3 where the bunch characteristics and phase space are plotted at different accumulated intensity (the intensity increases during the observation because the injection is still in progress). It should be noted that shape of these curves depends on accidental factors and is a subject of statistical variation. However, it is seen that, in any case, the induced voltage and other parameters of the bunches undergo the perturbation due to the beam instability which threshold can be estimated as 4×10^{12}.

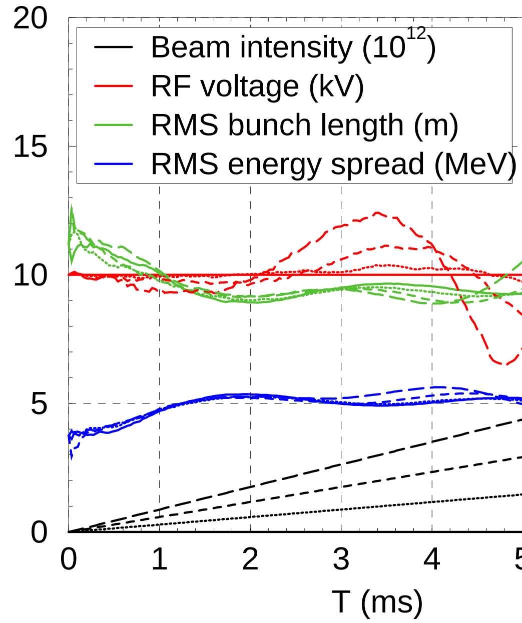


Figure 2: Evolution of beam parameter in the AR at different beam intensity. Solid lines – no loading, dotted lines – final intensity $N = 2 \times 10^{12}$, dashed lines $N = 4 \times 10^{12}$, long dashed lines – 6×10^{12}. Shape of the curves depends on accidental factors and may vary. Substantial enhancement of the perturbation is observed at $N >\sim 4 \times 10^{12}$.

Radical improvement of the situation is obtainable with feedback system added. Skeleton diagram of the system which was applied at the simulation is shown in Fig. 4. The results allow to conclude that combination of the feed-forward and the wide-band feedback systems is capable to prevent the beam loading and instability effects at all, so that beam intensity 2.1×10^{14} can be achieved, and final (at $t = 6.8$ ms) distribution looks therewith just as it is shown in Fig. 3-top.

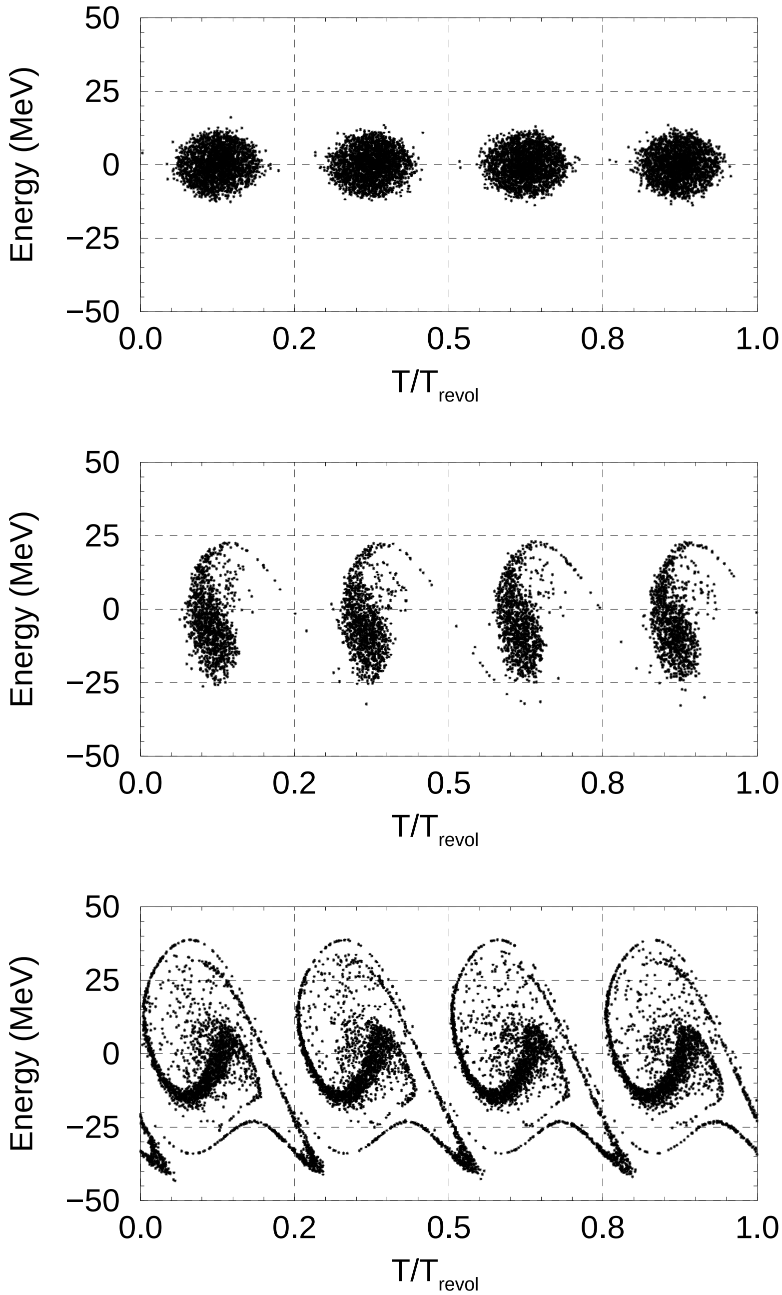


Figure 3: Phase space of accumulated beam with loading and feed-forward (no feedback). Top: $N = 0$, middle $N = 4 \times 10^{12}$, bottom $N = 6 \times 10^{12}$

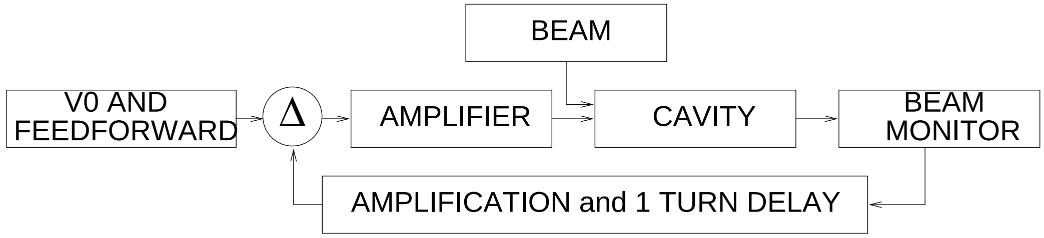


Figure 4: Skeleton diagram of feedback system.

4 Compressor Ring

In the bunch Compressor Ring, cavities are identical with the Accumulator Ring but 12 cavities should be applied to reach project voltage. Therefore overall impedance of the cavities is 600 kΩ, and beam induced voltage could reach about 19 MV in the collider mode with designed intensity (4 times

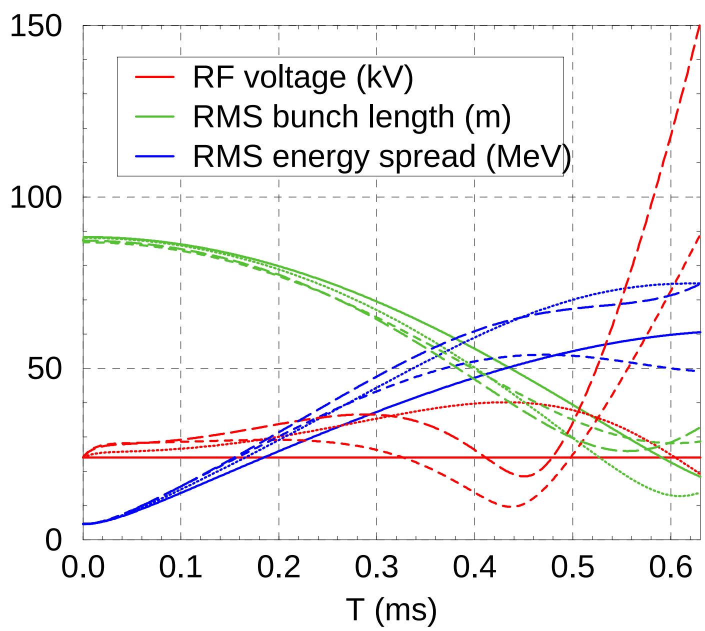


Figure 5: Beam parameters at bunch compression. Solid lines – zero intensity, dotted lines – $N = 2.5\times10^{13}$, dashed lines 4×10^{13}, long dashed lines – 5×10^{13}.

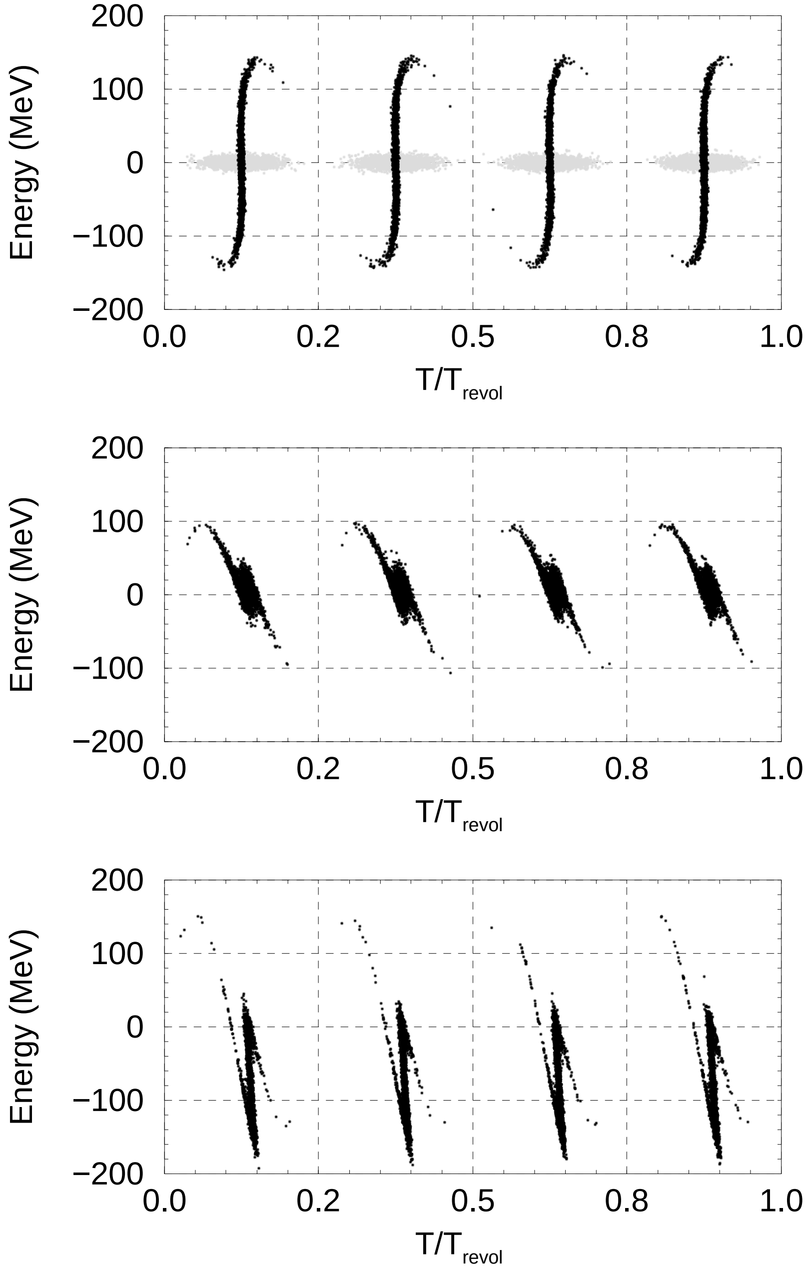


Figure 6: Phase space of beam in the Compressor Ring with loading and feed-forward effects. Top: $N = 0$ (initial and final distribution), middle $N = 4 \times 10^{13}$, bottom $N = 5 \times 10^{13}$

less in the NF mode). The feed-forward programming which is undeniably required in these conditions does not solve the problem entirely because of beam instability. Some examples are presented in Fig. 5-6 where the bunch characteristics and the beam phase space are presented at different intensity. Though accidental (statistical) factors affect the beam behavior, one can conclude that unacceptable large perturbations can appear at beam intensity about 10^{13} particles which is less of designed intensity in order of value. However, feedback system which skeleton diagram is shown in Fig. 4 allows to prevent the instability and to obtain really the same beam distribution as it is shown in Fig. 6-top.

5 Collider (HF): instabilities

Effects in the HF collider have the same genesis and similar manifestation as in above considered stages. However, as it follows from Table 3, relative beam loading in the collider is two orders of value less so the effects should be significantly weaker. Indeed, it was established by simulation that feed-forward system alone is sufficient to afford the bunches lifetime 10 ms at number of particles $N = 2 \times 10^{12}$ in each of $\mu^{\pm}$ bunches. True, threshold of the instability occurs rather close to that so the lifetime is halved at $N = 3 \times 10^{12}$. However, the instability is easy suppressible by the feedback system sketched in Fig.4, which allows to raise the threshold in two orders of value.

6 Collider: Potential well distortion

Another beam loading effect which may be important in the Collider Ring is potential well distortion due to high pulse current and rich harmonic composition of a short bunch. We assume that the beam-induced RF harmonic (long term effect) is compensated with help of feed-forward and feedback systems as it has been described in previous sections. Then the main harmonic has a rated voltage V_0, and short term perturbation appears only with the bunch passing through the cavity, resulting this total voltage

$$V = -\frac{V_0\eta}{|\eta|}\sin\phi - \sqrt{\frac{L}{C}}\int_0^{\phi} J(\phi')\cos(\phi-\phi')\,d\phi' \tag{1}$$

where phase ϕ is measured from synchronous value at constant reference energy E, η is slippage factor, L and C are the cavity inductance and capacity, J is the beam current. Restricting the consideration to a short bunch, $|\phi| \ll 1$, one can write equations of small synchrotron oscillations in the form

$$\frac{d\epsilon}{d\theta} = -\frac{eV_0\eta\phi}{2\pi|\eta|E} - \frac{e}{2\pi E}\sqrt{\frac{L}{C}}\int_0^\phi J(\phi')\,d\phi' \tag{2}$$

$$\frac{d\phi}{d\theta} = \frac{h\eta\epsilon}{\beta^2} \tag{3}$$

where θ is longitudinal coordinate (azimuth), ϵ is relative energy deviation from reference value E, h is RF harmonic number. Corresponding equation of phase trajectory is

$$\frac{W(\phi)}{\hat{\beta}_0^2} + \frac{\epsilon^2}{2} = \frac{\epsilon_0^2}{2} \tag{4}$$

with longitudinal beta-function

$$\hat{\beta}_0 = \sqrt{\frac{2\pi h|\eta|E}{eV_0\beta^2}} \tag{5}$$

(subindex '0' emphasizes that this parameter is determined without beam loading and can vary with the loading). Potential energy W which appears in Eq. (4) and further is determined by the equation

$$W(\phi) = \frac{\phi^2}{2} + \frac{|\eta|}{\eta V_0}\sqrt{\frac{L}{C}}\int_0^\phi J(\phi')(\phi-\phi')\,d\phi' \tag{6}$$

However, the beam current $J(\phi)$ in unknown function in this relation because it depends on the beam loading and should be determined from the equations as well. We will use the fact that steady state distribution of the bunch can be an arbitrary function of energy of synchrotron oscillations which is presented by left hand part of Eq. (4). Taking Gaussian distribution as an important and easy-to-analysis case, one can present the beam current in the form:

$$J(\phi) = J(0)\exp\left(-\frac{W(\phi)}{\sigma^2}\right) \tag{7}$$

where σ is distribution parameter coinciding with the rms bunch length when the beam loading is neglected. Substituting it in Eq. (6) and differentiating, one can get following equation for the potential:

$$\frac{d^2W(\phi)}{d\phi^2} = 1 + A\exp\left(-\frac{W(\phi)}{\sigma^2}\right) \tag{8}$$

with A as a loading parameter:

$$A = \frac{\eta J(0)}{|\eta| V_0}\sqrt{\frac{L}{C}} \tag{9}$$

The equation has the first integral

$$\frac{1}{2}\left(\frac{dW}{d\phi}\right)^2 = W + \sigma^2 A\left[1 - \exp\left(-\frac{W(\phi)}{\sigma^2}\right)\right] \tag{10}$$

that is

$$\frac{dW}{d\phi} = \pm\sigma\sqrt{2}\sqrt{\frac{W}{\sigma^2} + A\left[1 - \exp\left(-\frac{W(\phi)}{\sigma^2}\right)\right]} \tag{11}$$

At $A > -1$, there are stable synchrotron oscillations, and Eq. (11) has a solution:

$$\frac{\phi}{\sigma} = \pm\sqrt{2}\int_0^{\sqrt{W}/\sigma}\left[1 + A\,\frac{1-\exp(-x^2)}{x^2}\right]^{-1/2} dx \tag{12}$$

It follows from this that $W(\phi)$ is an even function of ϕ which is plotted in Fig. 7 in normalized terms at different A. It follows from the figure that for small oscillations

$$W(\phi) \simeq (1 + A)\,\frac{\phi^2}{2}, \qquad |\phi| \ll \sigma \tag{13}$$

Now we need to consider phase trajectory passing through the point $\phi = 0$, $\epsilon = \epsilon_0$. Using Eq. (4) one can represent area inside the trajectory in the form

$$S = 4\int \epsilon(\phi)\,d\phi = 4\int_0^{\phi_{max}}\sqrt{\epsilon_0^2 - \frac{eV_0E\beta^2}{\pi h|\eta|}\,W(\phi)}\;d\phi \tag{14}$$

where integration is performed over the part of first quadrant of the phase space where the under-root expression is positive Further, Eq. (10) can be

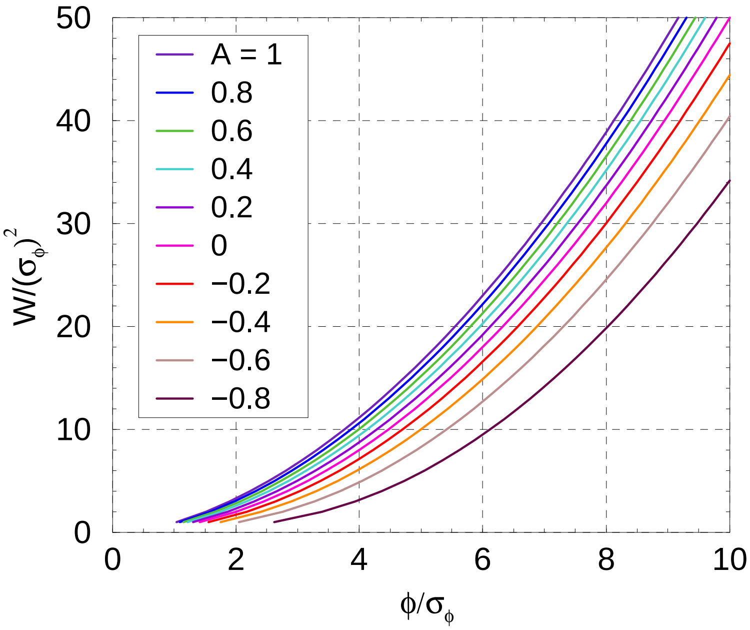


Figure 7:

used to reduce Eq. (14) to the form

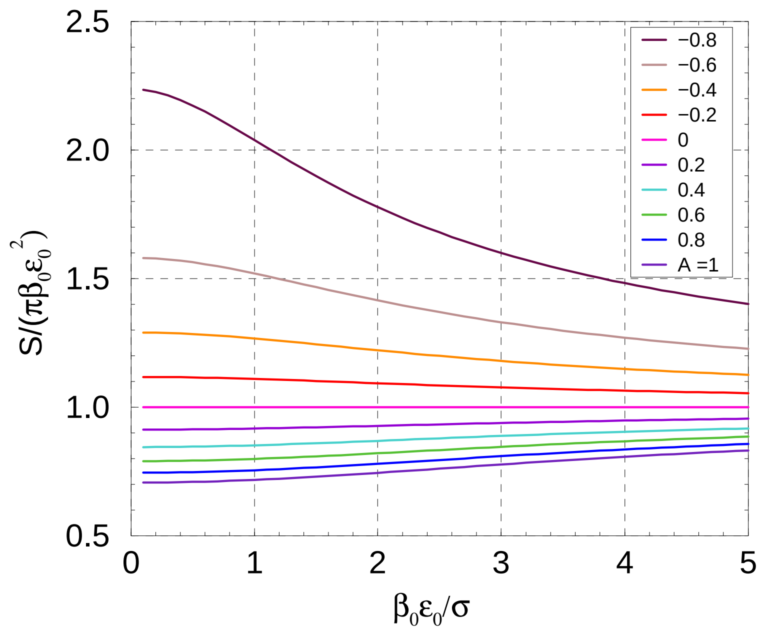
$$S = \frac{4\epsilon_0}{\sigma\sqrt{2}} \int_0^{\hat{\beta}_0^2\epsilon_0^2/2} \frac{\sqrt{1 - W/(\hat{\beta}_0^2\epsilon_0^2/2)}\; dW}{\sqrt{W/\sigma^2 + A\,[1 - \exp\,(-W/\sigma^2)]}} \tag{15}$$

The substitution $W = \xi^2\hat{\beta}_0^2\epsilon_0^2/2$ allows to get more convenient expression

$$\frac{S}{\pi\hat{\beta}_0\epsilon_0^2} = \frac{4}{\pi}\int_0^1 \left[1 + A\,\frac{1 - \exp(-\xi^2\hat{\beta}_0^2\epsilon_0^2/2\sigma^2)}{\xi^2\hat{\beta}_0^2\epsilon_0^2/2\sigma^2}\right]^{-1/2} \sqrt{1 - \xi^2}\; d\xi \tag{16}$$

Note that $\pi\hat{\beta}_0\epsilon_0^2$ is area of the phase trajectory passing through the point $(0,\, \epsilon_0)$ *without beam loading*. Therefore, Eq. (16) describes relative change of phase area depended on energy deviation and the loading parameter, as it is plotted in Fig. 8. Approximately, it is the ratio of loaded/unloaded bunch lengths and longitudinal beta functions:

$$\frac{S}{\pi \hat{\beta}_0 \epsilon_0^2} \simeq \frac{\phi_{max}}{\phi_0} = \frac{\hat{\beta}}{\hat{\beta}_0} \tag{17}$$



Following parameters of HR ring can be obtained from Tables 1-3:

$$\sqrt{L/C} = R/Q = 50\,\Omega, \quad J(0) = 680\,\mathrm{A}, \quad \hat{\beta}_0 = 7900, \quad A = 0.34$$

Therefore, the loading effect is very inessential in the case: longitudinal beta function decreases by 14% at small synchrotron oscillations, and by 7% at large amplitude ($5\,\sigma$ level).

7 Conclusion

Beam induced accelerating voltage is very large in the Accumulator Ring and in the Compressor Ring so that its suppression with help of feed-forward and feed-back systems is required to reach projected beam intensity. Similar systems are needed in the HF collider ring as well, though the beam loading effects are significantly less there. Potential well distortion is negligible in practice.

References

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