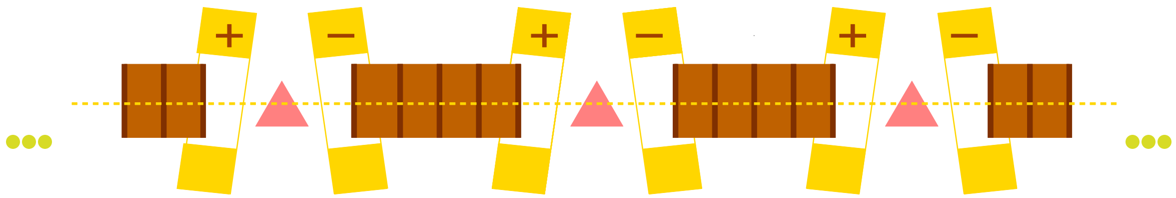
Rectilinear 6D cooling channel with half-flip cells and tilted solenoids

V. Balbekov, MAP meeting 05/31/2013

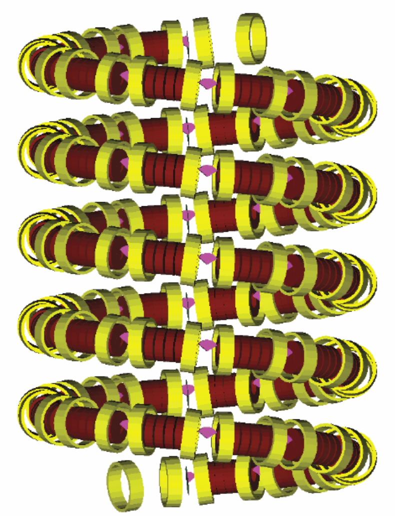
Any 4D cooling channel with alternating solenoids
can be transformed into rectilinear 6D cooling channel
by small alternating tilt of the solenoids and using of wedge absorbers

Rectilinear FOFO snake (from MAP meeting 02/01/2013)

An inclination of the solenoids at alternating angle 20-50 mrad and adding of wedge absorbers allows to transform FOFO of RFOFO 4D cooling channel into 6D cooling channel

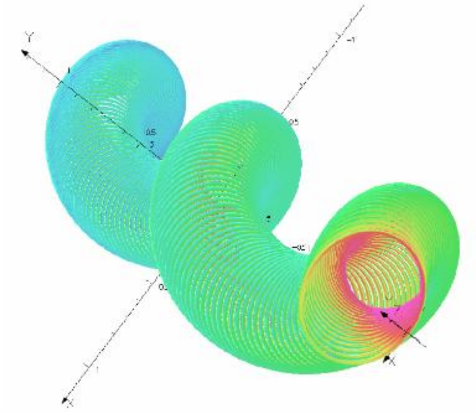


Guggenheim

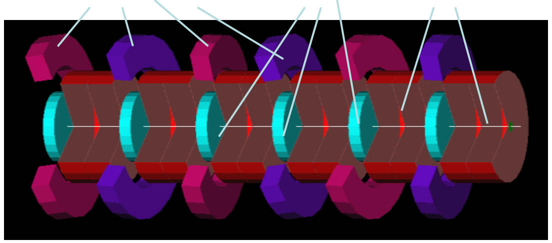


This R_FOFO channel has about the same performance as other known 6D cooling channels with comparable field, being simpler in construction.

Helical Cooling Channel

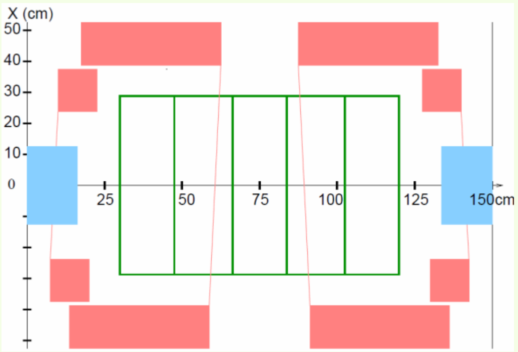


Helical FOFO snake

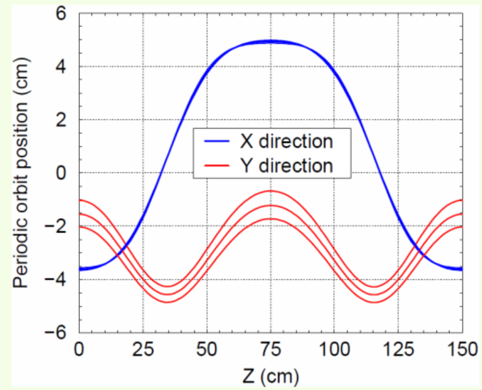


Example: transformation RFOFO to R_FOFO (MAP 02/01/2013)

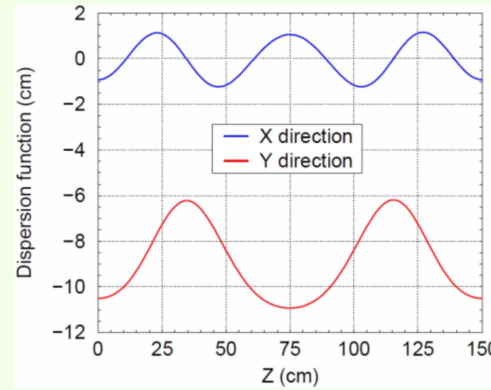
Solenoid tilt 50 mrad

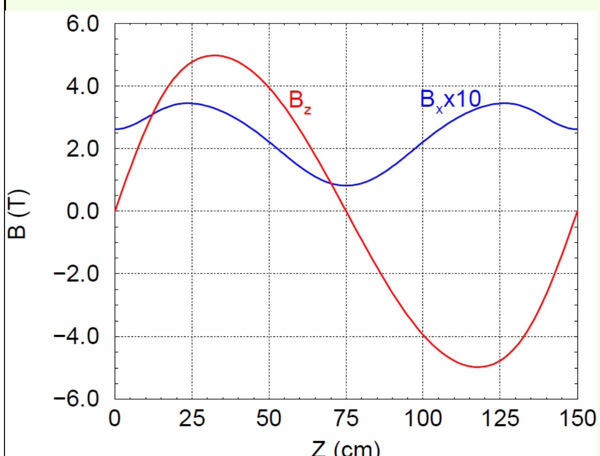


Periodic orbits

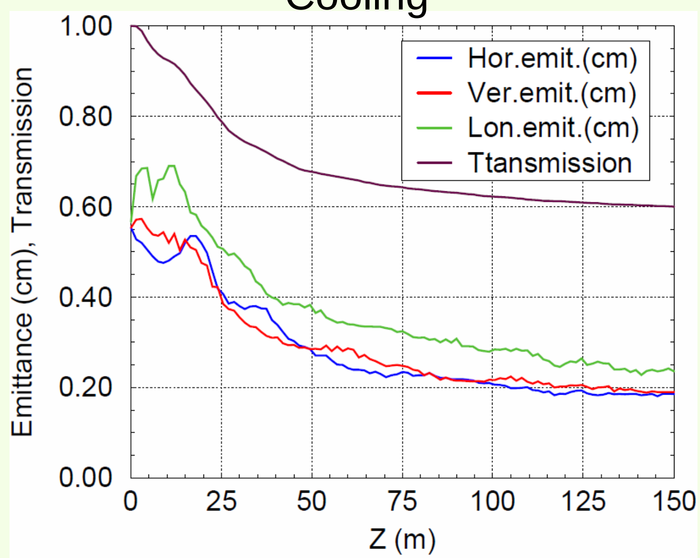


Dispersion functions





Cooling



Maximal field in the coil is 7.7 T

~2 mm emittances are achievable

Half-flip channel: recent proposals

Half Flip 6D Lattice

R. B. Palmer, Rick Fernow
(BNL)

Thursday

2/14/13

Essence

- Half flip solenoid cells are used for ultimate 4D cooling of muons in a long tapered channel
- Matrices are used for emittance exchange to estimate applicability of the channel for 6D cooling

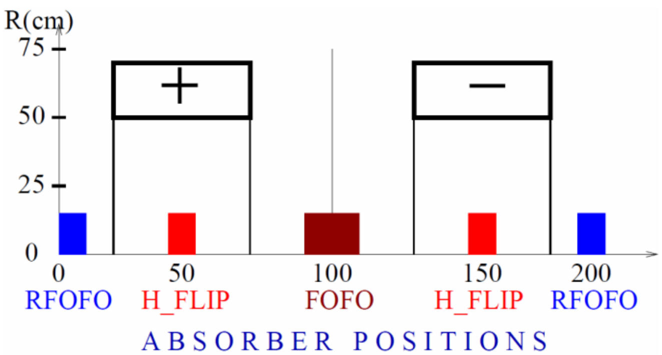
My goal is to use tilted solenoids

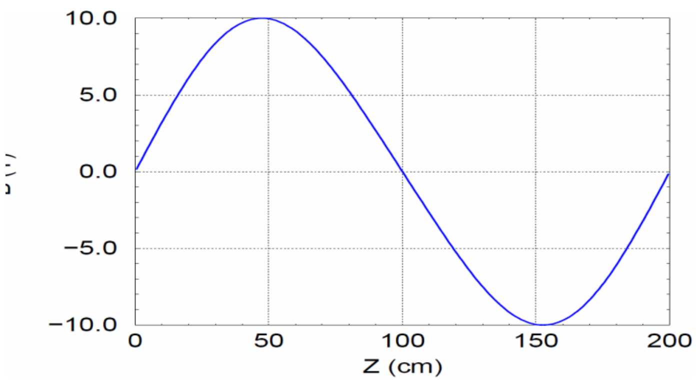
to transform this 4D cooling channel to 6D channel

How does Half-Flip cell differ from FOFO and RFOFO channel?

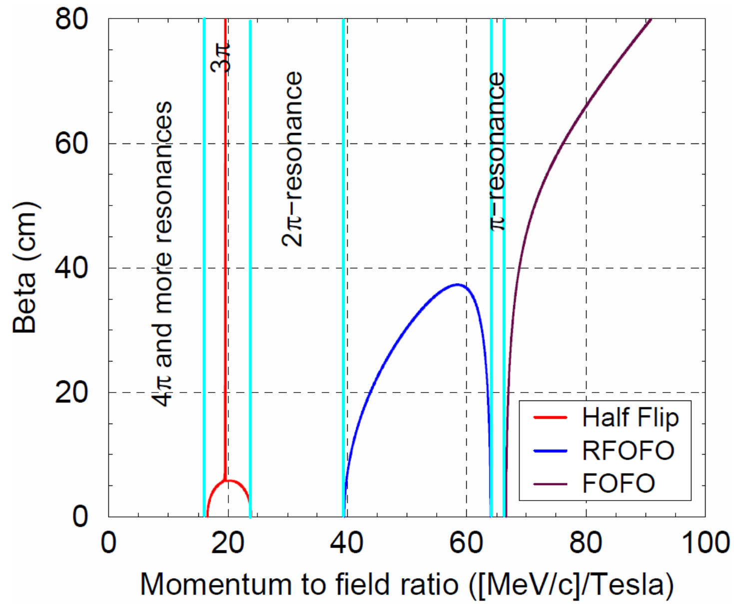
Actually it is the same alternate solenoid channel using with different beam momenta

Example: a cell schematic and field





Minimal beta vs momentum-to-field ratio.
3 regions of stability separated by resonances are shown.
(position of β_{min} depends on the region).



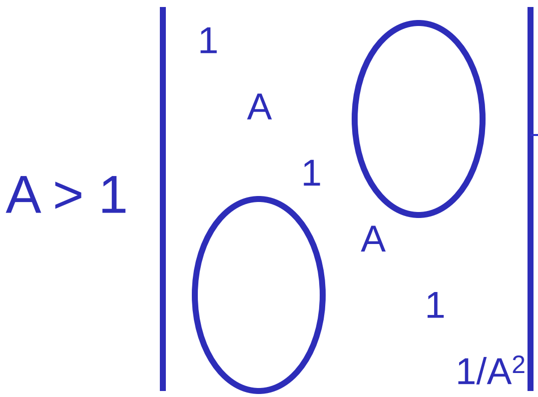
Thus the H-F cell is about the same as FOFO or RFOFO cells
but with higher phase advance due to lower momentum to field ratio.

It is characterized by less beta-function and less momentum acceptance
which is divided into 2 parts by 3π resonance if the left-right symmetry is broken

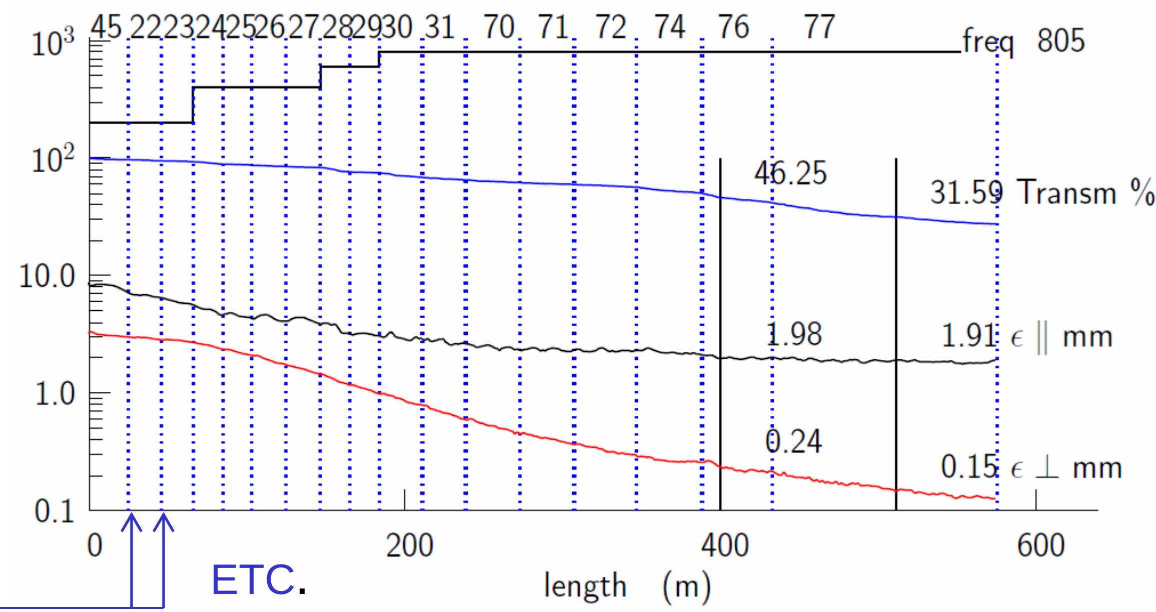
4D cells + matrices (by B. Palmer and R. Fernow)

This tapered channel has length about 600 m and uses 77 kinds of cells.

Beam evolution is shown with emittance-exchange by matrices



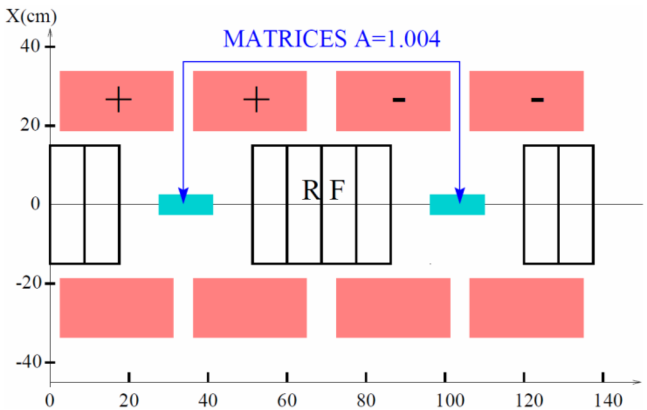
ICOOL using matrices for half-flip with longer cells

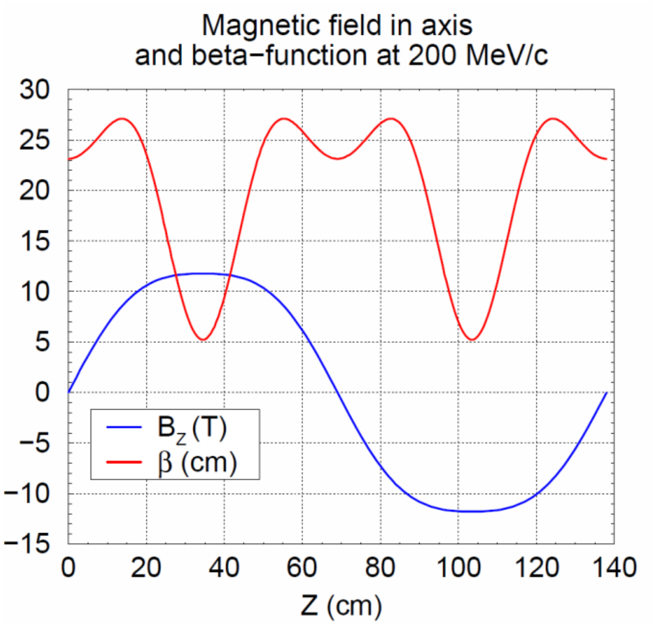


file	β cm	cell cm	L cm	dL cm	R cm	dR cm	j A/mm^2
70	5.2	68.75	3.000	28.000	18.000	15.000	117.26
71	4.6	68.75	0.000	29.000	18.000	15.000	105.77
72	3.9	68.75	0.000	13.000	12.000	15.000	96.80
			13.000	16.000	18.000	15.000	96.85
74	2.9	58	4.218	8.436	5.905	21.091	158.14
			12.655	6.327	19.404	7.593	134.22
76	2.1	58	1.687	10.967	4.218	16.873	153.79
77	1.6	58	0.000	10.967	4.218	16.873	158.75

Cell #70 is taken for the study with tilted solenoids (A=1.004)

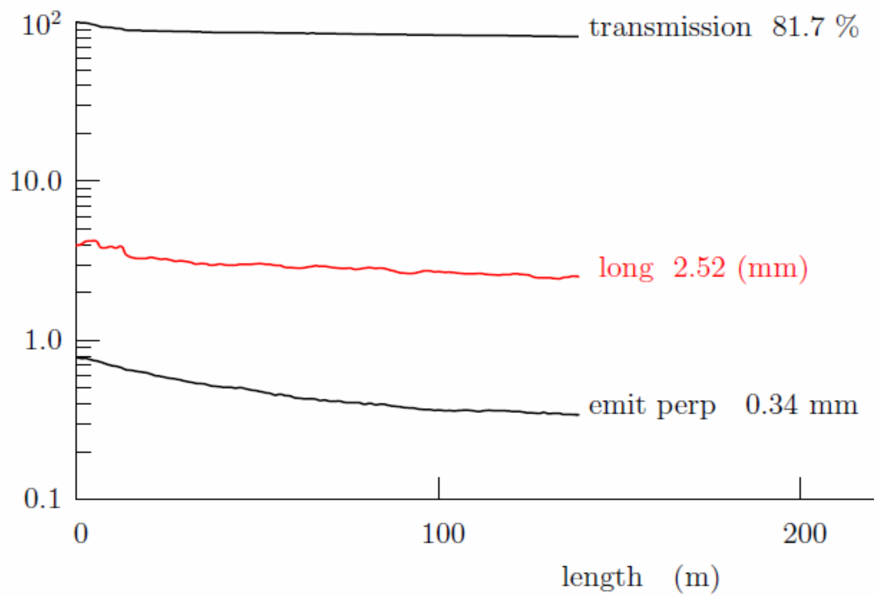
Cooling channel with cells #70

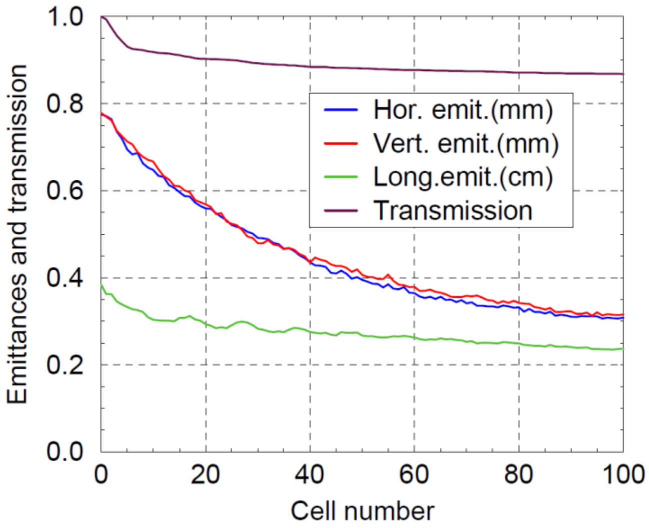




Period length	137.5 cm
Maximal field strength on axis:	11.8 T
Maximal field strength in coil	13.4 T
Current density	117 A/mm^2
Reference momentum	200 MeV/c
Accelerating frequency	805 MHz
Accelerating gradient	22.7 MV/m
Synchronous phase	25°
LH_2 absorber thickness	10.8 cm
Minimal beta-function	5.2 cm

Comparison of simulations (no tilts, no wedges, matrices)





<u>Top:</u> B.P. data

<u>Bottom:</u> My data

<u>After 100 cells</u>:

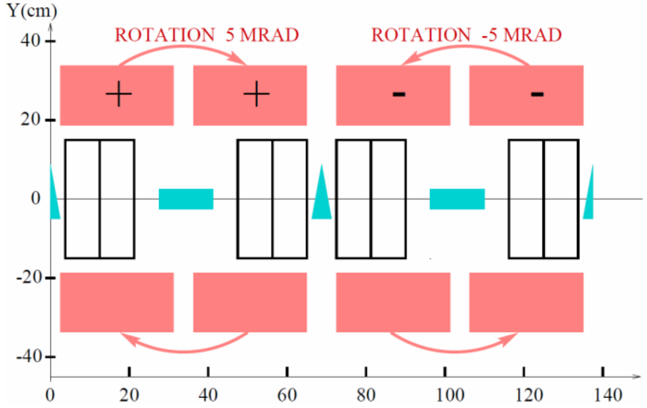
B: E_{trans}= 0.34 mm, E_{long}= 2.52 mm, Tr.= 81.7%

M: E_{trans}= 0.32 mm, E_{long}= 2.37 mm, Tr.= 86.8%

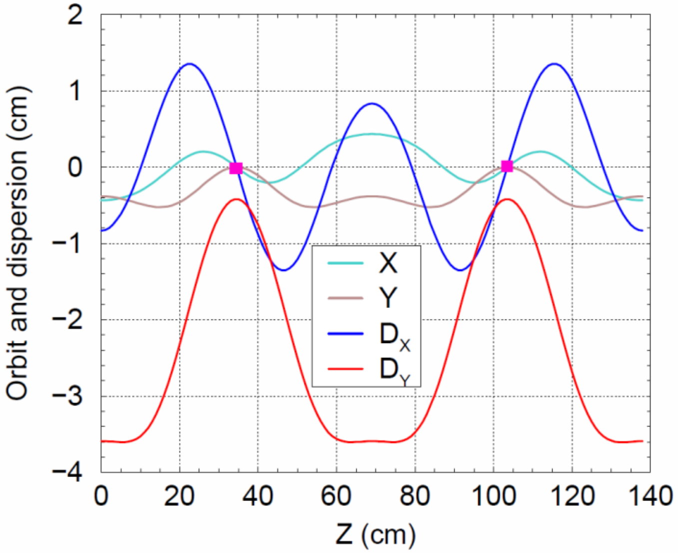
- Emittances – good agreement.
- Transmission is sensitive to matching of initial distribution with the channel, especially to betatron amplitude – total momentum correlation (I have presented my best result).

Transformation of 4D half flip cell to 6D one (tilt 5 mrad)

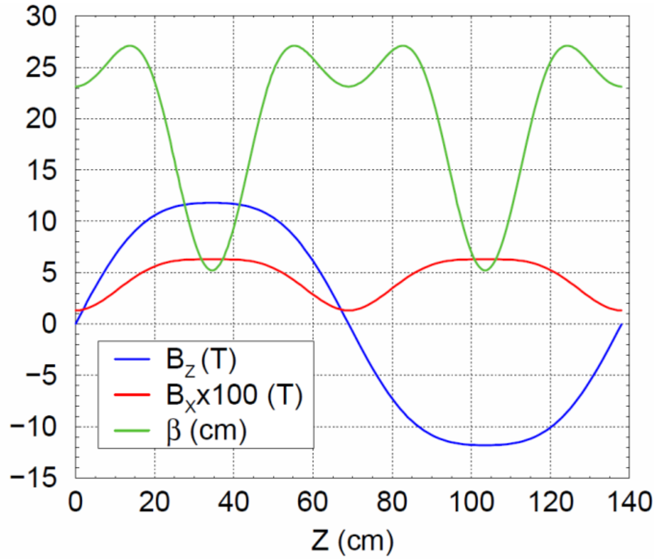
Schematic of the transformation
Solenoids are tilted   5 mrad horizontally



Periodic orbit and dispersion



Field and beta-function with tilt

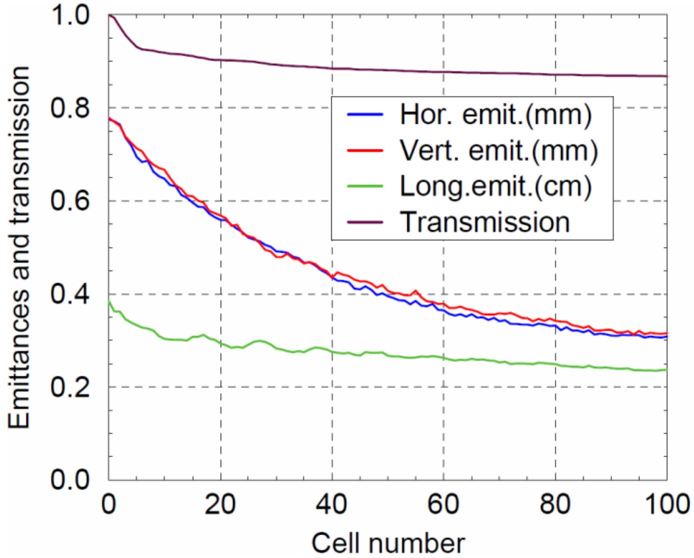


Z = 35 cm and Z = 103 cm are appropriate points ■ for planar absorbers (minimal beta-function, maximal value and proper sign of derivatives X and D_x)

However, the derivatives are not enough in value to provide sufficient emittance exchange in this cell, with tolerable solenoid tilt.

Therefore, I use wedge absorbers as they are shown in the picture

Cooling simulations with matrices; P = 200 MeV/c

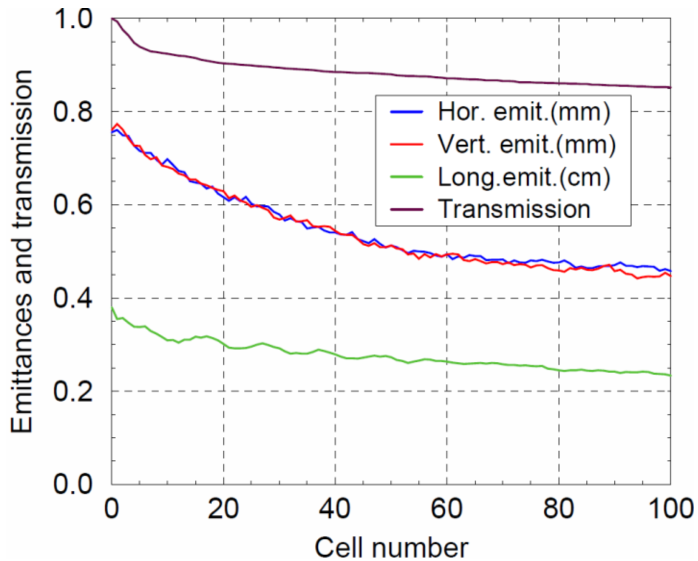


<u>Upper plot:</u> Basis cells, no solenoid tilt, planar absorbers of thickness 10.8 cm at β=5.2 cm

Emittance exchange by matrix at absorber centers

<u>After 100 cells</u>:

E_{trans}= 0.32 mm, E_{long}= 2.37 mm, Tr.= 86.8%



<u>Lower plot:</u> Two kinds of planar absorber are used: 9.2 cm long at β=5.2cm and 1.6 cm at β=23cm

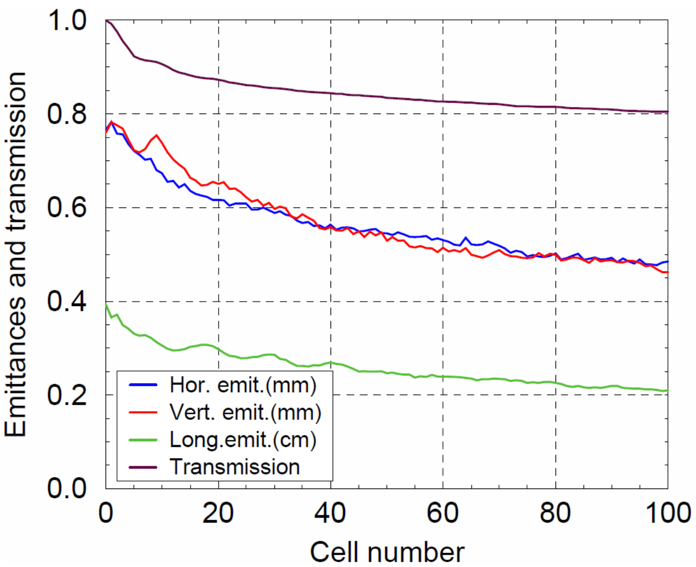
Emittance exchange by matrices is applied

<u>After 100 cells</u>:

E_{trans}= 0.45 mm, E_{long}= 2.33 mm, Tr.= 85%

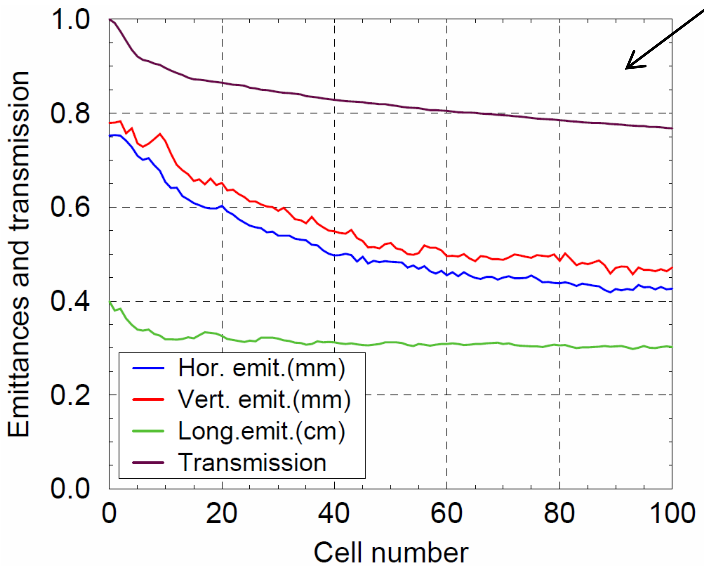
The absorbers at higher β result in 40% increasing of transverse emittance.

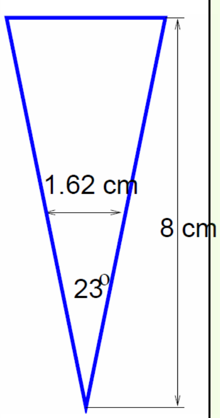
Cooling simulations with 5 mrad tilted solenoids



Upper plot
Two planar absorbers, tilted solenoids, emittance exchange by matrices
After 100 cells:
E_{trans}= 0.47 mm, E_{long}≈ 2.1 mm, Tr.≈ 80%

Lower plot:
Emittance exchange by wedge absorber
After 100 cells:
E_{trans}= 0.44-0.48 mm, E_{long}≈ 3 mm, Tr.≈ 76%





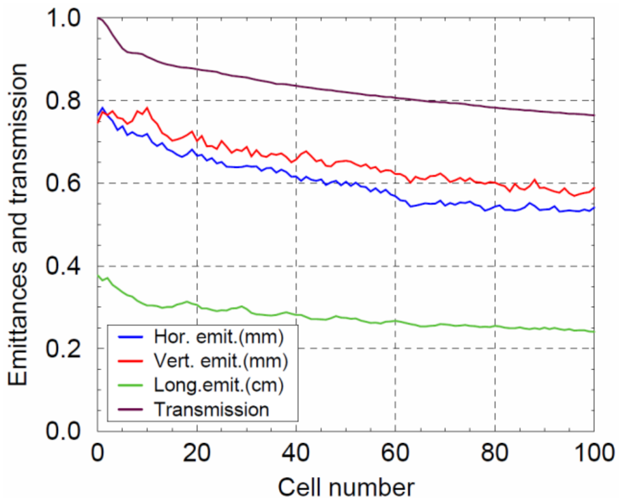
Emittance exchange is insufficient with these wedge absorbers and soleniod tilts resulting in higher longitudinal emittance and additional partice losses in comparison with matrices.

A thickening of wedge absorbers and/or increase of solenoid tilt suggests itself.

However …

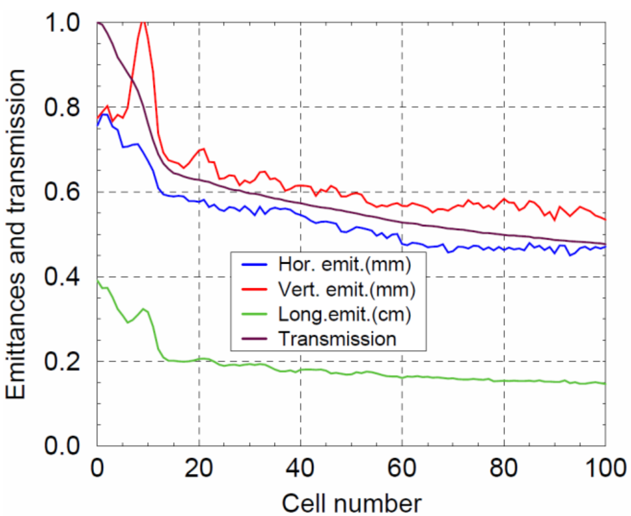
Cooling simulations with thicker wedge absorbers or higher tilt

Abs. 8.1+2.7 cm, tilt 5 mrad

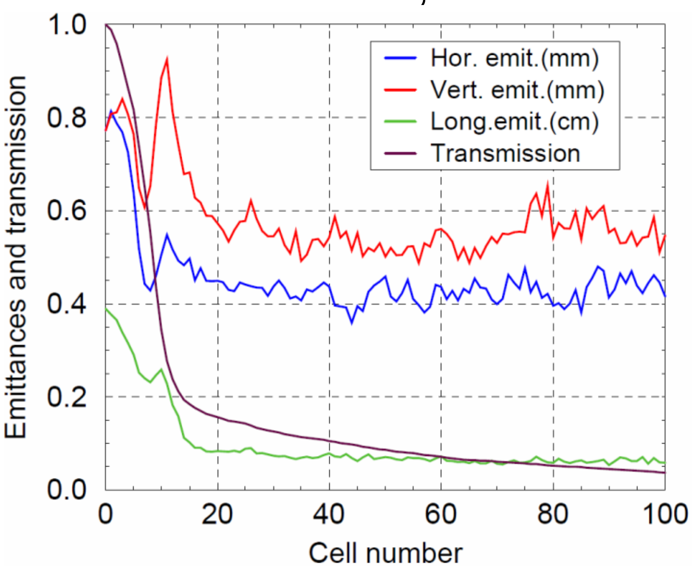


- Thicker wedge absorber results in higher transverse and 6D emittance (left top graph)
- 7 mrad tilt looks not bad a little redistributing emittances.
- But larger tilt results in scraping accompanied by decrease both longitudinal emittance and transmission (left bottom graph for 10 mrad).
- 15 mrad tilt destroys the beam at all (right bottom graph)

Abs. 9.2+1.6 cm, tilt 10 mrad



Abs. 9.2+1.6 cm, tilt 15 mrad



RESULTS

Probably, transverse emittance about 0.5 mm,

and longitudinal emittance about 3 mm

are near ultimate parameters of the beam

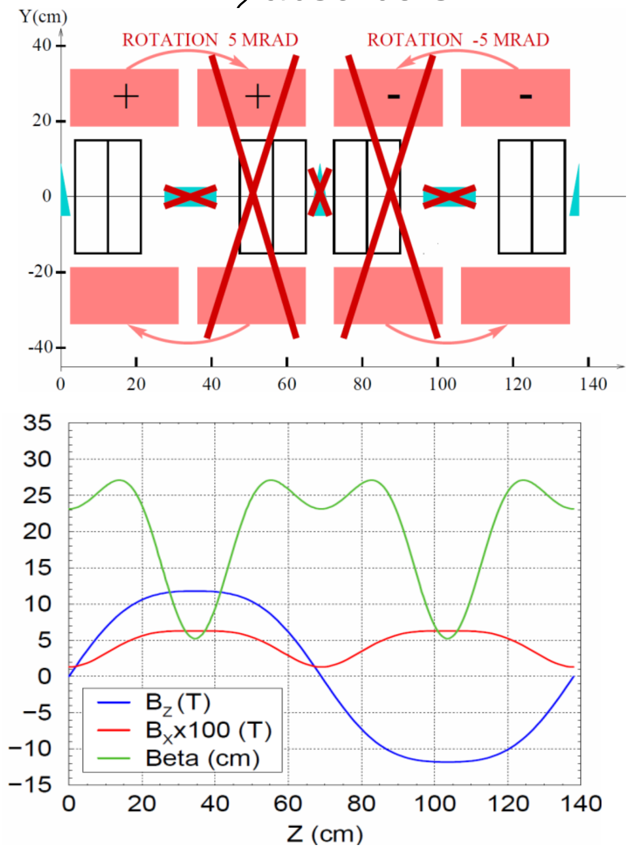
with this kind of half-flip cell (138 cm, 11.8 T).

However, it is reasonable to compare this with cooling by R_FOFO cells with closely related parameters

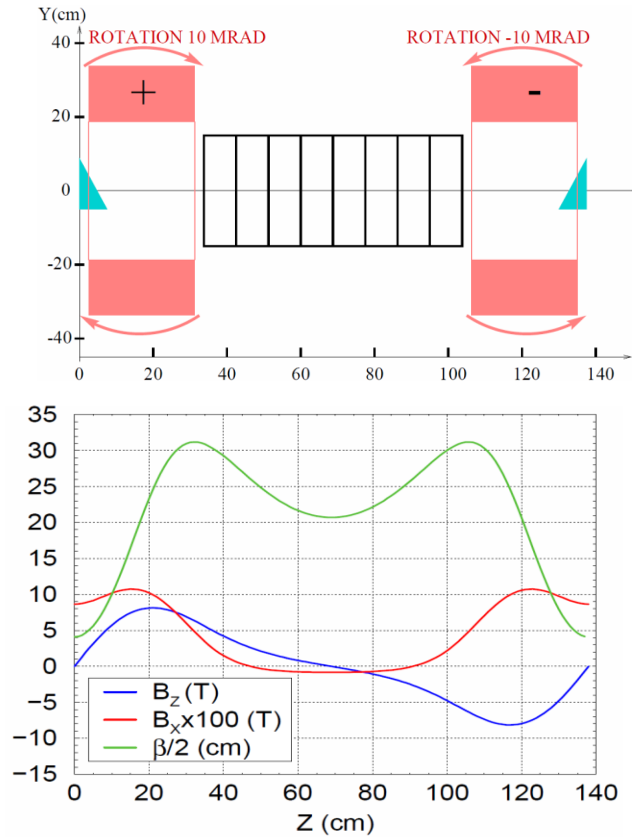
The simplest way is to remove part of the coil

Removal of 2 coils from the HalfFlip cell transforms it to R_FOFO

LH_2 absorbers



LiH absorbers

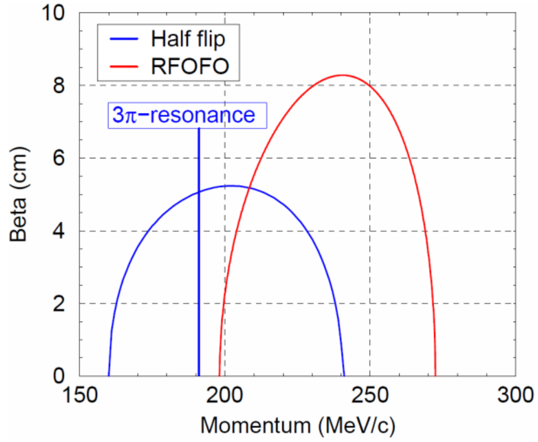


LH₂ absorbers	LiH absorbers
Maximal field on axis 11.8 T	Maximal field on axis 8.13 T
Maximal field on coil 13.4 T	Maximal field on coil 11.4 T
Minimal beta-function 5.2 cm	Minimal beta-function 8.2 cm

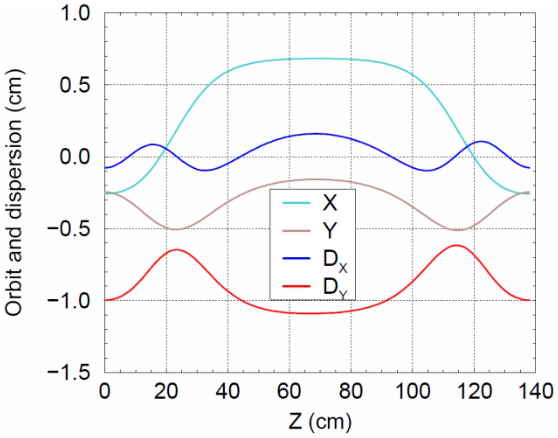
Effective beta-functions are almost equal in value)

The R_FOFO performances (not optimized)

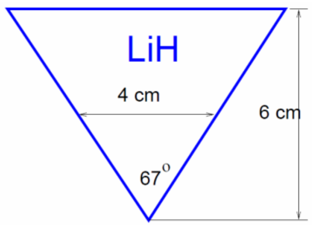
Beta-functions vs momentum
R_FOFO is compared with HF

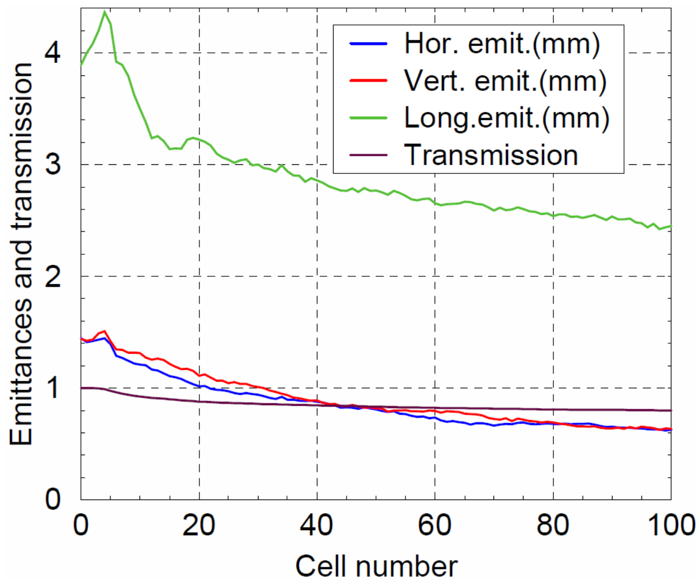


Periodic orbit and dispersion



LiH absorber





- HF and RFOFO have about the same momentum acceptances but HF region is divided by 3π-resonance which grows with tilt.
- Dispersion of this RFOFO cell is not so much but sufficient for emittance exchange with LiH wedge absorbers.
- Achievable emittances are about 0.6 mm transversely and 2.4 mm longitudinally at transmission 80% (w/o decay).

The R_FOFO with decreased field and momentum

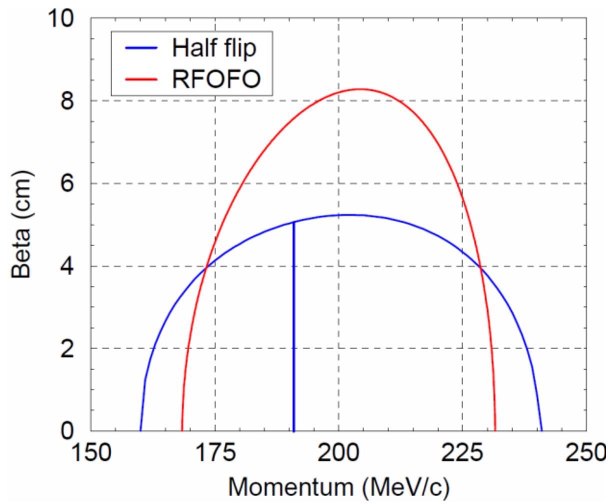
Rather close beam parameters obtained with proportionally decreased momentum and field:

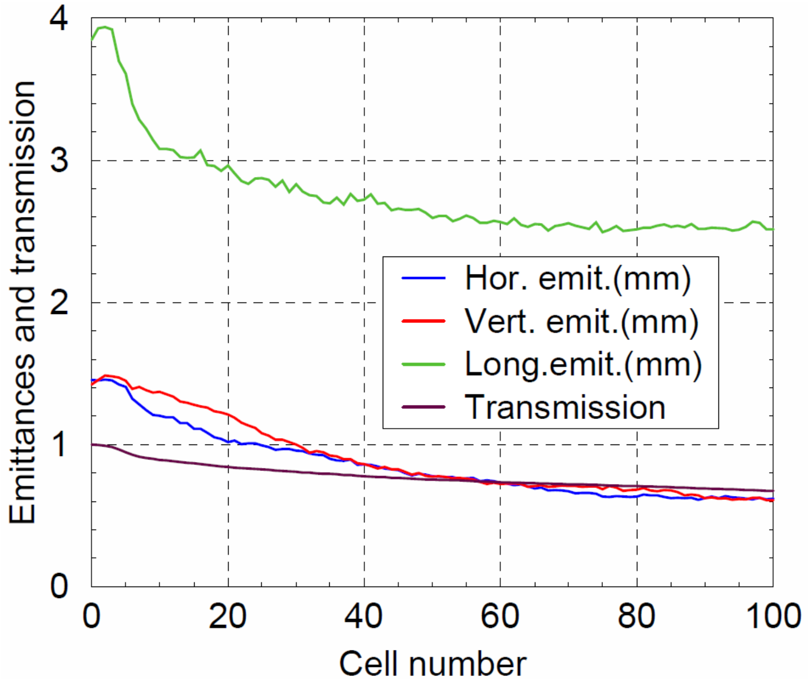
$P = 200$ MeV/c, $j = 100$ A/mm^2, $B_{axis} = 6.9$ T, $B_{coil} = 9.7$ T,

$\varepsilon_{trans} = 0.61$ mm, $\varepsilon_{long} = 2.5$ mm, Transmission 68%

Hopefully, transverse emittance about 0.4 mm can be obtained with axial field about 10 T.

Beta-functions vs momentum
(decreased R_FOFO field)





CONCLUSION

- Likewise other alternate solenoid cooling cells, half flip cell can be converted from 4D mode into 6D mode by inclination of the solenoids in some plane.
- Distinction of the half flip cell is that maximum of dispersion function appears in other position than minimum of beta function. Therefore, separated wedge absorbers are applied in combination with flat main absorbers. Their thickness has to be limited because of higher beta function
- Applicable tilt angle is limited as well due to 3π resonance which arises inside the half-flip working area at the tilt.
- Therefore, power of the emittance exchange is restricted in the half flip cells. However, with fitting parameters, the power is sufficient to reach a noticable longitudinal cooling.
- The cell of length 137.5 cm with maximal axial field 11.8 T, 5-7 mrad tilt, and LH_2 absorber allows to reach transverse emittance 0.45-0.5 mm and longitudinal one 3-2.5 mm.

CONCLUSION (continued)

- Partial removal of coils converts the half flip cell into R_FOFO cell.
  In such a case, maximal field decreases from 11.8 T to 8.13 T in axis,
  and from 13.4 to 11.4 T in the coils.
- Achievable emittances in the R_FOFO cells with LiH absorbers are
  0.6 mm transversely and 2.3 mm longitudinally.
  With higher field, transverse emittance 0.4-0.5 mm looks quite reasonably.

- Therefore, it is an open question which cell is better.

  R_FOFO requires less superconductor.

  However, HF cell with planar absorbers would be suitable for simultaneous cooling of both sign muons. This case is unrealizable with considered short – high field cell but more studies is required for longer cells.

Appendix: simulation with planar absorbers (tilt 10 mrad)