Beam stability for Higgs Factory parameters

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Collective effects for the Fermilab Higgs Factory design [1] are analyzed. Resistive wall impedance is assumed, the aperture radius b is taken as 4.5 of the beam rms size σ_x.

Transverse plane

The growth rate of the head-tail horizontal mode number l can be estimated by the air-bag model [2]:

$$\tau_l^{-1} = -\frac{N_\mu r_0 c}{4\pi\gamma T_0 \omega_x} \int_{-\infty}^{\infty} d\omega \,\mathrm{Re}\, Z_x(\omega) J_l^2\left(\omega\frac{\hat{z}}{c} - \chi\right) .$$

Here all the notations are identical to the reference; $Z_x(\omega)$ is the total transverse impedance, ring-averaged with the beta-function:

$$Z_x(\omega) = \frac{\omega_y}{c} \oint Z_x(\omega, s)\beta_x(s) ds .$$

For a thick resistive wall, the local impedance scales as $Z_x(\omega, s) \propto 1/b^3(s) \propto 1/\beta_x^{3/2}$, resulting in a weak dependence of the total impedance contributions on the beta-function:

$$Z_x(\omega) \propto \oint \frac{ds}{\sqrt{\beta_x(s)}} \cong C\sqrt{\frac{Q_x}{R}} .$$

Taking all the parameters as in Ref [1], $N_\mu = 2\cdot 10^{12}$; $Q_x = 4.56$; $Q_y = 3.56$; $Q_s = 0.002$; $\eta = 0.08$; $\Delta p_{\mathrm{rms}} / p = 3\cdot 10^{-5}$; tungsten wall with the conductivity $\sigma_W = 1.8\cdot 10^{17}\,\mathrm{s}^{-1}$, for the chromaticity $\xi = 5$ one gets the growth rate

$$\tau_1^{-1} \cong 0.5\cdot 10^{-5}\omega_0$$

where $\omega_0 = 6.3\cdot 10^6\,\mathrm{rad/s}$ is the angular revolution frequency.

To estimate the geometrical impedance, one can assume its longitudinal value $Z_{\|} / n = 0.1\text{Ohm}$; after application of the Panofsky-Wenzel theorem, this leads to doubling of the total transverse impedance. In the result, one may have only a few percent of growth of an initial perturbation after 1000 turns, so there is a safety factor about one hundred for the transverse plane instabilities.

Longitudinal plane

To estimate how significant could be longitudinal collective effects, a relative change of the synchrotron frequency due to the potential well distortion can be computed [3]:

$$\Delta\omega_s = \frac{N_\mu r_0 c\eta}{2\pi\omega_s \gamma C} \int_0^\infty d\omega \exp\left(-\omega^2\sigma_\tau^2 / 2\right) \omega \,\mathrm{Im}\, Z_{\|}(\omega)$$

with $\sigma_\tau = \sigma_s / c$ as the rms bunch length in seconds. With the described model of the resistive wall impedance, this leads to

$$\frac{\Delta Q_s}{Q_s} \approx 0.2 \frac{N_\mu r_0 \eta R^2 \bar{\delta}}{Q_s^2 \gamma \sigma_s^3 \bar{b}},$$

where $\bar{\delta} = c / \sqrt{2\pi\sigma_W / \sigma_\tau}$ is the skin depth at the bunch frequency $\omega = \sigma_\tau^{-1}$, and $\bar{b} = 4.5\sqrt{R\varepsilon_n / (\gamma Q_x)}$.

For the normalized rms emittance $\varepsilon_n = 0.2\text{mm}$, this yields $\Delta Q_s / Q_s \approx -0.6$. With the mentioned geometrical impedance, this number doubles. That high number of the potential well distortion means that one should expect up to ~30% of the energy widening. This effect could be probably reduced by means of the second harmonic RF.

References

[1] Y. Alexahin, Preliminary Design of the mu+mu- Higgs Factory Ring Lattice, MAP-doc-4351-v1 (2012).

[2] A. Chao, “Physics of collective beam instabilities in high energy accelerators”, Eq. (6.213), p. 350, J. Wiley & Sons, 1993.

[3] Ibid, Eq. (6.58), p. 291.